EXAM RULES:

- No books, notes, or other study aids except for a calculator provided by the math department. You may not use your own calculator.
- Phones must be turned off. If your phone rings during the test, you will automatically lose 5 points.
- Sending or receiving text messages during the exam constitutes cheating and will be treated as such.
- No restroom breaks until you have turned in your exam.
- On sections where you have a choice, clearly mark the problem you want graded, otherwise, this choice will be made for you.
1. (6 pts) **Definitions and Theorems.** Give formal statements of the following definitions and theorems. (For (a) and (b), “formal” means involving limits; or, for (a), you may give the $\epsilon$-$\delta$ definition instead.)

(a) A continuous function.

(b) The derivative of a function $f(x)$ at the point $x_0$.

(c) A polynomial of degree $n$.

(d) A rational function.

(e) The First Fundamental Theorem of Calculus.

(f) The Second Fundamental Theorem of Calculus.
2. (4 pts) **Limits.** Find the limits if they exist. If a limit does not exist, write DNE.

(a) \[ \lim_{x \to -1} \frac{x^2 + 5x + 4}{x^2 + 3x + 2} \]

(b) \[ \lim_{t \to 6} \frac{6}{36 - t^2} \]

(c) \[ \lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \]

(d) \[ \lim_{t \to \infty} \frac{(2t^2 + 3t - 2)(5t - 1)}{10t^3 + 4t - 2000} \]
3. (4 pts) **Derivatives.** If possible, compute the derivative at the given point $a$. If the derivative doesn’t exist at $a$, explain why.

(a) $f(x) = 2x^5 - \pi + \frac{2}{x^5}; \quad a = 1.$

(b) $f(x) = |x|; \quad a = 0.$

(c) $y = x^7 \sin x; \quad a = \pi/2.$

(d) $y = \ln(x^2 + 2x + 1); \quad a = -1$
4. (4 pts) **Integrals.** Compute the following integrals.

(a) \[ \int_{1}^{2} x + \frac{1}{x} \, dx \]

(b) \[ \int_{1}^{5} x \sqrt{x} - 1 \, dx \]

(c) \[ \int x e^{5x} \, dx \]

(d) \[ \int \frac{x + 1}{x^2 - 4} \, dx \]
5. (2 pts) Implicit differentiation can be used to find the equation of the tangent line to the curve $xy^3 + xy = 12$ at the point $(6, 1)$. The equation of this tangent line can be written in the form $y = mx + b$. Find $m$ and $b$.

6. (2 pts) Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 8 square miles per hour. How rapidly is the radius of the spill increasing when the area is 11?

7. (2 pts) The population of numbats in Western Australia is given (in millions) by $\frac{dn}{dt} = 0.1n - 0.02n^2$. In the year 2000 (count this as year zero) the population was 3 million. Find the population in year $t$. 