Some Important Limits

In this lab we will investigate some limits which we shall need later. Note that they are all on the \( \frac{0}{0} \) variety.

1. \[ \lim_{x \to 0} \frac{\sin x}{x} \]
   a. Enter the expression \( \frac{\sin x}{x} \) in Maxima. Now evaluate the expression at \( x = 1/2 \) as follows: Highlight the expression and click on substitute under the Simplify menu. A dialog box with three blanks will appear. Put “x” in the substitute box and 1/2 in the “with” box. “\( \sin(x)/x \)” should already be in the “in” box but beware of an extra “;”. Now click ok. Maxima will return “2 \( \sin(2) \)”, which is correct, but not too useful. Highlight this answer and click “to float” in the Numeric menu. Now the answer comes out as a decimal. Another way to accomplish is to substitute .5 instead of 1/2 in the original expression.
   
   b. You can substitute several values in the expression at once by entering:
      \[
      \text{makelist}(\sin(x)/x, x, [1, .5, .1, .01, .001, .0001])
      \]
      Do it.
   
   c. Us your computations above to guess \( \lim_{x \to 0} \frac{\sin x}{x} \).
   
   d. Now find the left hand limit \( \lim_{x \to 0^-} \frac{\sin x}{x} \).

   e. Finally, graph the function \( \frac{\sin x}{x} \) and zoom in on the graph at \( x = 0 \) to confirm your answers to parts c and d.

2. Investigate the limit \( \lim_{x \to 0} \frac{e^x - 1}{x} \) as you did above. Note: To evaluate the exponential function, you can enter exp(x) or \((\%e)^x\). You cannot use the letter e alone.

3. Now investigate the limit \( \lim_{x \to 0} \frac{2^x - 1}{x} \). This one is harder only because you will probably not recognize the answer.