Warm-up Exercise (done by T.A.) Homing pigeons avoid flying over large bodies of water, preferring to fly around them instead. Assume that a pigeon released from a boat 1 mi from the shore of a lake flies first to point $P$ on the shore and then along the straight edge of the lake to reach its home at $L$. If $L$ is 2 mi from point $A$, the point on the shore closest to the boat, and if a pigeon needs $10/9$ as much energy per mile to fly over water as over land, find the location of point $P$, which minimizes energy used.

Exercise 1. To pass a lifeguard test at Waikiki beach, a lifeguard must get from a point, $A$, at the water’s edge to a buoy in the water at point $B$ in the shortest possible time. Suppose the buoy is exactly 30 m off-shore. Let $C$ be the point on the beach closest to the buoy and suppose $C$ is exactly 40 m from where the lifeguard stands at point $A$. The lifeguard wants to know the best point $P$ on the beach at which to jump into the water and start swimming. Suppose her running speed is 3 meters per second and her swimming speed is 1 meter per second.

a. Draw a picture of the scene and label it.

b. Find a function $T(x)$ that gives the time it takes to get from point $A$ to point $B$, given an input value $x$ (for example, you might decide that $x$ should represent the distance from $C$ to $P$).

c. Find the value of $x$, and hence the point $P$, that will minimize the time it takes to reach the buoy.
Exercise 2. You have just been hired as a consultant by The Coca-Cola Company. Your first assignment is to decide whether the current can design is the most efficient one possible. Of course, they will continue to produce cans of Coke containing 12 fluid ounces, or 355 cm³. That is, the volume of the can must be 355 cubic centimeters.

a. Find the dimensions (radius of base and height) of the can which uses the least amount of aluminum. (Hint: volume of a cylinder is equal to the area of the base times the height.)

b. Of course, soft drink manufacturers account for other things besides the cost of aluminum, and current can designs typically have a base radius of approximately 3.25 cm and a height of about 12 cm. Given these dimensions, how much aluminum do cans currently use? (You can ignore the fact that the dimensions of such cans don’t appear to hold 12 ounces.)

c. If Coca-Cola decides to adopt your new can design, how much aluminum will they save per can?