Exercise Part I. Recall your recent homework problem:

18. A rectangle is to be inscribed under the arch of the curve $f(x) = 4 \cos(x/2)$, for $-\pi \leq x \leq \pi$. What are the dimensions of the rectangle with largest area, and what is the largest area?

a. Explain why the max value occurs at a point $x_0$ in the interval $[0, \pi]$ such that $\cot(x_0/2) = x_0/2$.

b. Use Maxima to plot the function $f(x) = \cot(x)$ and $g(x) = x$ on the same graph.
   Draw the graph here:

c. Use this graph to find an approximate solution in the interval $[0, \pi]$.
   (Hint: you want the value $x_0$ in $[0, \pi]$ which satisfies $f(x_0/2) = g(x_0/2)$.)
Exercise Part II. Recall from Lab 4, Newton’s Method provided a way to find approximate solutions to equations of the form $f(x) = 0$. The procedure was as follows: first make a guess, $x_1$, and find the point $(x_1, f(x_1))$ on the graph of $f$, which corresponds to $x_1$. Then draw the tangent line to the graph at that point and follow the tangent line down to the x-axis. Label the point of intersection $x_2$. This is your new guess. It should be better than the old one.

The formula for $x_2$ in terms of $x_1$ is:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$  

We repeated this process using $x_2$ as the starting point to produce a third guess $x_3$, and then a fourth and on and on. After not too many steps, we saw that we could often (but not always!) find an approximate solution to the equation $f(x) = 0$.

Use Newton’s method to solve the equation $\cot(x/2) = x/2$.

a. Define your own guess function in Maxima that is appropriate for this equation. Write the function definition here:

b. Make an initial guess based on your plot from Part I, and use your guess function to compute guesses 2, 3, 4, etc. Write your guesses here:

c. Continue until the new guess, $x_n$, is almost the same as the old guess, $x_{n-1}$, and write this final guess here:

d. Check that your answer for part c. really is an approximate solution. (For example, if your answer is the number $x_n$, you might use Maxima to compute the difference $\cos(x_n/2) - x_n/2$ – or maybe the relative difference $(\cos(x_n/2) - x_n/2)/(x_n/2)$.) Write down what evidence you found to support your answer to part c.

e. Finally, finish (approximately) solving the optimization problem from Part I.