Chapter 4

- What is a random variable?
  - Formal definition of a random variable.
  - Functions of random variables.
  - Distribution and Expectation
Example 1.

- Let \( \Omega \) be a human population containing \( n \) individuals.

\[
\Omega = \{ \omega_1, \omega_2, \ldots, \omega_n \} \quad (4.1.1)
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  $A(\omega) =$ the age of individual $\omega$. 
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- Thus to each $\omega$ is associated a number $A(\omega)$, the number of years person $\omega$ has lived.
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- integer-valued vs. real-valued functions.
Similarly, we may denote the height, weight, and income by the functions
\[ \omega \mapsto H(\omega), \]
\[ \omega \mapsto W(\omega), \]
\[ \omega \mapsto I(\omega). \]
For some medical purposes, a linear combination of height and weight may be a useful measure:
\[ \omega \mapsto \lambda H(\omega) + \mu W(\omega), \]
where \( \lambda \) and \( \mu \) are two numbers. This is also a function of \( \omega \).
Similarly, if \( \omega \) is a "head of family," the census bureau may consider the function:
\[ \omega \mapsto I(\omega) N(\omega), \]
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Persons between ages 20 and 40:

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or simply \( \{ 20 \leq A \leq 40 \} \).

Describe the set of all persons with height between 65 and 75 (inches) and weight between 120 and 180 (pounds).
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More examples of random variables

- **Example 2.**
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- **Example 3.**
  \[ \Omega = \text{a collection of } n\text{-dimensional vectors.} \]
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Definition (Random Variable)

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Comments on the term “random variable” vs. alternatives:

- “random quantity”
- “stochastic variable”
The adjective "random" reminds us that $X$ is not just an ordinary function. It is a function of a sample space $\Omega$. Thus, $X$ describes some feature of a random event or chance phenomenon.

Important point: What is random about $X(\omega)$ is the sample point $\omega$, which is picked at random from the sample space $\Omega$. Once $\omega$ is picked, $X(\omega)$ is determined and there is nothing vague, indeterminate, or chancy about it.

For instance, after a particular apple $\omega$ is picked from the bushel $\Omega$, its weight $W(\omega)$ can be measured and may be considered as a known fixed quantity (not random).

Standard notation for random variables: $X(\omega) = x$. 

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What’s so random about a random variable?

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Proposition 1
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Proposition 2
If $\varphi : X \times Y \to \mathbb{R}$ is a function of two variables then...

Example
Let $\varphi(x, y) = \sqrt{x^2 + y^2}$, and suppose $X(\omega)$ and $Y(\omega)$ denote the horizontal and vertical velocities of a gas molecule $\omega \in \Omega$; then $\varphi(X(\omega), Y(\omega)) = \sqrt{X^2(\omega) + Y^2(\omega)}$ denotes the absolute speed of $\omega$. 
Functions of random variables

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denotes the absolute speed of $\omega$. 
Important points about functions of rv’s

Proposition 1 is a special case of Proposition 2. Proposition 2 handles the case $f(X) = \phi(X,Y)$.

Proposition 2 generalizes to functions $\phi(x_1,...,x_n)$ of $n$ variables:

$\phi: X_1 \times \cdots \times X_n \to \mathbb{R}$

Important Example

The sum of $n$ random variables:

$\phi(X_1(\omega),...,X_n(\omega)) = X_1(\omega) + \cdots + X_n(\omega)$

We will denote this function by $S_n(\omega)$. 

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Chapter 4

What is a random variable?
Formal definition of a random variable.
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Distribution and Expectation
Let $X$ be a (real-valued) random variable. For two real numbers $a \leq b$, define the event 
\[ A = \{ a \leq X \leq b \} = \{ \omega \in \Omega | a \leq X(\omega) \leq b \} \].

Is this really an event? What is $P(A)$?

\[ P(A) = P(a \leq X \leq b) = P(\{ \omega | a \leq X(\omega) \leq b \}) \].
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What is $P(A)$?

$$P(A) = P(a \leq X \leq b) = P(\{ \omega \mid a \leq X(\omega) \leq b \}).$$
More generally, let $\mathbb{R}$ be a set of real numbers. For example, $\mathbb{R} = [0, 2] = \{ x \in \mathbb{R} | 0 \leq x \leq 2 \}$, and consider the event $\{ X \in \mathbb{R} \}$. Then, $P(\{ X \in \mathbb{R} \}) = P(\{ \omega | X(\omega) \in \mathbb{R} \})$.

If $\mathbb{R} = [0, 2]$, then $P(\{ X \in \mathbb{R} \}) = P(\{ \omega | X(\omega) \in [0, 2] \}) = P(0 \leq X \leq 2) = P(\{ \omega | 0 \leq X(\omega) \leq 2 \})$.

What if $\mathbb{R}$ is a single number, say, $\mathbb{R} = \{ 1.5 \}$?
More generally, let $R$ be a set of real numbers. For example,

$$R = [0, 2] = \{x \in \mathbb{R} \mid 0 \leq x \leq 2\},$$

and consider the event $\{X \in R\}$. 

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Then,

$$P(X \in R) = P(\{\omega \mid X(\omega) \in R\}).$$
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What if $R$ is a single number, say, $R = \{1.5\}$?
Suppose $X$ is a random variable with range of possible values \{ $x_1, x_2, \ldots, x_n$ \}. That is, for each $\omega \in \Omega$, we have $X(\omega) = x_k$ for some $k = 1, 2, \ldots, n$.

Suppose we know the values $p_k = P(X = x_k)$ for each $k = 1, 2, \ldots, n$. Then, for any subset $R \subset \{x_1, \ldots, x_n\}$, we can compute $P(X \in R) = \sum_{x_k \in R} P(X = x_k) = \sum_{x_k \in R} p_k$. 
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\[P(X \in R) = \sum_{x_k \in R} P(X = x_k) = \sum_{x_k \in R} p_k.\]
Distribution Function of a Random Variable

When $R = (-\infty, x]$, we define $F_X(x) = P(X \in R) = P(X \leq x) = \sum_{x_k \leq x} p_k$.

For two real numbers $a \leq b$, we have $P(a < X \leq b) = P(X \leq b) - P(X \leq a) = F_X(b) - F_X(a)$.

Can you see why?

Hint: Describe the event \{X \leq b\} as a disjoint union.
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Mathematical Expectation

Let $X$ be a random variable defined on a countable sample space $\Omega$. 

**Definition**
The mathematical expectation of $X$ is the number

$$E(X) = \sum_{\omega \in \Omega} X(\omega) P(\{\omega\}),$$

provided this series converges absolutely.

Think “weighted average,” with weights $P(\{\omega\})$.

**Example**
As above, let $X$ be a rv with values $\{x_1, x_2, \ldots, x_n\}$, and let $p_k = P(X = x_k)$.

$$E(X) = \sum_{\omega \in \Omega} X(\omega) P(\{\omega\}) = n \sum_{k=1}^{n} x_k p_k.$$
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Examples of Mathematical Expectation

- *Waiting time* for a head to appear in a sequence of tosses of a (biased) coin.
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- *Waiting time* for a head to appear in a sequence of tosses of a (biased) coin.
- *Waiting time* for a red to appear in a sequence of spins of a roulette wheel.

\[
X = \text{number of spins until red appears.}
\]

\[
p = P(\text{red}) = \frac{18}{38} \approx 0.47
\]

\[
q = P(\text{not red}) = \frac{20}{38} \approx 0.53
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P(X = n) = q^{n-1}p
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Chapter 4: Random Variables

July 1, 2011
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The probability of getting red on the $n$th spin is:

$$P(X = n) = (q \cdots q)p = q^{n-1}p$$

The expected value $E(X)$ is:

$$E(X) = \sum_{n=1}^{\infty} n P(X = n) = \frac{1}{p}$$