Mailbox

An interval in the subgroup lattice of a finite group which is isomorphic to \( M_7 \).

WALTER FEIT

The study of congruence lattices of finite universal algebra leads to the study of intervals in the subgroup lattices of finite groups. See [1] especially p. 14 or [2] Theorem 2. I am indebted to János Kollár and Peter P. Pálfy who first brought these results to my attention.

For any natural number \( n \) let \( M_n \) denotes the lattice of length 2 with \( n \) atoms.

Let \( H \) be a subgroup of the finite group \( G \). The interval \([H, G]\) in the subgroup lattice of \( G \) is isomorphic to \( M_n \) for some \( n > 0 \) if and only if there are exactly \( n \) subgroups \( K \) with \( H \subseteq K \subseteq G \) and each of these is a maximal subgroup of \( G \).

It is an interesting question to determine those \( n \) such that \( M_n \) is isomorphic to some \([H, G]\).

If \( G \) is solvable and \([H, G]\) is isomorphic to \( M_n \) for some \( n > 2 \) then it is known that \( n = 1 \) is a prime power. See [2] Theorem 3. The object of this note is to give an example that shows in contrast to the solvable case that \( M_7 \) is isomorphic to \([H, G]\) for suitable \( G \) and \( H \).

Let \( G \) be the alternating group on 31 letters. Let \( H \subseteq G \) with \(|H| = 31.5 \) and let \( N = N_G(H) \). Thus \(|N| = 31.5 \).

\( SL_2(2) \) acts on 31 points and 31 hyperplanes in the projective 4 space over \( F_2 \). Thus there exist \( K_1, K_2 \) with \( H \subseteq K_i \subseteq G \) and \( K_i = SL_2(2) \) but \( K_i \) not conjugate to \( K_j \) in \( G \). For \( i = 1, 2 \)

\[ \{K_i^x \mid x \in G, H \leq K_i^x \} = \{K_i^x \mid x \in N \}. \]

Thus \(|\{K_i^x \mid x \in G, H \leq K_i^x \}| = 3\).

---

\( ^1 \) This work was partly supported by NSF Grant MCS-8201333.

Presented by H. P. Gumm. Received July 19, 1982. Accepted for publication in final form September 30, 1982.

REFERENCES


Yale University
New Haven, Conn.
U.S.A.