Math 304 - Lab Exercise 4

Due Thursday, October 29 2009

Consider the nondimensional chemostat model:

\[
\frac{dn}{dt} = \frac{\alpha}{1+c} n - n, \quad \frac{dc}{dt} = -\frac{c}{1+c} n + c + \beta
\]

where \(\alpha > 0\), and \(\beta > 0\). There are steady states at \((0, \beta)\), and \((n^*, c^*)\), where

\[n^* = \alpha(\beta - \frac{1}{\alpha - 1}), \quad c^* = \frac{1}{\alpha - 1}.
\]

The stability of the steady states and the existence of the nontrivial steady state in the first quadrant depends on the size of the parameters \(\alpha\) and \(\beta\).

**Exercise:** In exercise 3, you were asked to solve this system of differential equations for several sets of parameter values and plot the solutions against time. In this exercise, you will plot the phase plane portrait for the chemostat model for \(\alpha = 2\) and \(\beta = 2\).

**Step 1:** Plot the vector field of the chemostat model for \(0 \leq n \leq 4\) and \(0 \leq c \leq 4\) using the commands \([x1,x2]=\text{meshgrid}(0:0.5:4, 0:0.5:4)\) and \(\text{quiver}(x1,x2,dx1dt,dx2dt)\), where \(dx1dt\) and \(dx2dt\) specify the vector field. Use the **hold on** command to plot the solutions on the same figure.

**Step 2:** Using the MATLAB function from Exercise 3 to compute the derivatives \(\frac{dn}{dt}, \frac{dc}{dt}\), use the differential equation solver **ode45** to solve the system of ODEs for \(t \in tspan = [0, 40]\), the initial values \(x0 = (n0, c0) = (3, 4)\), and the parameter values: \(\alpha = 2, \beta = 2\). To call the ODE solver use the command:

\[
[t,x] = \text{ode45}(@\text{chemostat},tspan,x0,[],alpha,beta);
\]

where "chemostat" is the name of the function that evaluates the derivatives from Exercise 3. The output of the ODE solver is the vector \(t\) of the times at which the solutions have been computed and the vector of solutions \(x = (n, c)\).

**Step 4:** Plot the trajectory \((n(t),c(t))\) on the phase plane. Use the plot command **plot(x(:,1),x(:,2))**.

**Step 5:** Repeat Steps 3 and 4 for other initial values. Be sure to plot enough trajectories to fully characterize the phase plane. Label the axes of the phase plane plot.

**Step 6:** What can you conclude about the stability of the steady states?

Turn in listings of the MATLAB functions, the commands that you used to create the plot and the phase plane plot.