Math 140  Lecture 12

Exam 2 covers Lectures 7 - 12. Study the recommended exercises.
Review area, circumference, volume formulas - inside front cover.
RECALL. The graphs of \( e^x \) and \( e^{-x} \).

\[ y = e^x \]
\[ y = e^{-x} \]

- Graph \( y = e^{x-1} - 1 \).
  - Give the domain, range, intercepts and asymptotes.

\[ x\text{-intercept: } x = 1 \]
\[ y\text{-intercept: } y = \frac{1}{e} - 1 \approx \frac{1}{2.7} - 1 \approx -.63 \]
\[ \text{hor. asym. } y = -1 \]
\[ \text{domain } (-\infty, \infty) \text{ range } (-1, \infty) \]

**Logarithms**

Assume \( b > 0, b \neq 1 \). Thus \( b^x \) is 1-1 and it has an inverse.

**Definition.** \( \log_b(x) \), the log of \( x \) to the base \( b \), is the inverse of the exponential function \( b^x \).

\[ \ln(x) \text{, the natural logarithm, } = \log_e(x) = \text{the inverse of } e^x. \]

Note, “\( \ln \)” is “el-n” not “one-n”, not “eye-n”.

Inverses act in opposite directions and inverses cancel. \( y = \log_b(x) \iff b^y = x \).
\[ \ln(e^x) = x \]
\[ e^{\ln(x)} = x \]

If we have \( \ln(b^x) \) instead of \( \ln(e^x) \), then \( \ln \) and \( b^x \) don’t completely cancel and \( \ln(e^x) = x \) becomes:
\[ \ln(b^x) = x \ln b. \]

**FACT.** \( e^0 = 1 \Rightarrow 0 = \ln(1) \).
\[ e^1 = e \Rightarrow 1 = \ln(e) \]

- Simplify to a rational.
\[ \log_5 5^{\sqrt{5}} = \log_5 5^{\sqrt{5}} = \log_5 5^{1/2} = \log_5 5^{3/2} = \frac{3}{2} \]
\[ \log_{\sqrt{2}} \left( \frac{1}{8} \right) = \log_{\sqrt{2}} \left( \frac{1}{2^3} \right) = \log_{\sqrt{2}} 2^{-3} = -3 \]
\[ \log_{\sqrt{2}} 2 \text{ 1st solve } 8^x = 2. (2^{3})^x = 2^1, 3x = 1, x = \frac{1}{3} \]
\[ \log_8 2 = \log_8 8^{1/3} = 1/3 \]

**Write in log form.** Either (method 1: easy problems) use the definition of log or (method 2: hard problems) take the appropriate log of both sides and cancel inverses.

- **\( 2^3 = 8 \)**
  - Method 1. Since \( 2^x \) and \( \log_2(x) \) are inverse, \( 3 = \log_2(8) \)

- **\( 5^{2x-1} = 6 \)**
  - Method 2. Take the log of both sides.
  \[ \log_5(5^{2x-1}) = \log_5(6) \text{ so } 2x - 1 = \log_5(6) \]

**Solve for \( x \), write the answer using logarithms.**

- **\( x^2 = 4 \)**
  \[ x = \pm \sqrt{\log_5(4)} \]

- **\( e^{3x+1} = 8 \)**
  \[ 3t + 1 = \ln(8) \]
  \[ 3t = \ln(8) - 1 \]
  \[ t = \frac{1}{3}(\ln(8) - 1) \]

- **\( 3^{2x} = 5^{x+1} \)**
  - Use natural logarithms and method 2.
  \[ \ln(3^{2x}) = \ln(5^{x+1}) \]
  \[ 2x \ln 3 = (x + 1) \ln 5 \]
  \[ x(2 \ln 3 - \ln 5) = \ln 5 \]
  \[ x = \ln 5/(2 \ln 3 - \ln 5) = \frac{\ln 5}{\ln 9/5} \]

Since they are inverses, the graph of \( \log_b(x) \) is the reflection of \( b^x \) across the major diagonal and \( \ln(x) \) is the reflection of \( e^x \) across the major diagonal and \( \ln(x) \).

For the graph of \( \ln(x) \)

- **x-intercept:** \( x = 0 \) \text{ hor. asym. none}
- **domain:** \( (0, \infty) \text{ range } (-\infty, \infty) \)

**Graph \( y = \ln(x+1) + 1 \).**

- **y-intercept:** \( \ln(0+1)+1 = \ln 1 + 1 = 0 + 1 = 1 \)
- **x-intercept:** \( \ln(x+1)+1 = 0 \iff \ln(x+1) = -1 \iff x + 1 = e^{-1} \iff x = 1/e - 1 \approx 1/(2.7) - 1 \approx -.63 \)
- **vert. asym.:** \( x = -1 \)
- **hor. asym. none**
- **domain:** \( (-1, \infty) \text{ range } (-\infty, \infty) \)

**Graph \( y = 1 - \ln(1-x) \).** refl. across y, right 1, refl. across x, up 1.

The 5 log properties of Lecture 13 are needed for Exam 2.