23. Show that the matrix \( A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \) is singular.

25. Find the inverse of each of the following matrices.
(a) \( A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix} \)

(b) \( A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \)

27. If \( A^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix} \) and \( B^{-1} = \begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix} \), find \((AB)^{-1}\).

29. Find a solution to the linear system \( AX = B \) where \( A \) is the matrix in 25(a) above and
(a) \( B = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \)

(b) \( B = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \)

37. Show that if \( A \) is nonsingular and \( AB = O_n \), then \( B = O_n \) where \( A, B \) are \( n \times n \).

39. Consider the homogeneous system \( AX = O \), where \( A \) is \( n \times n \). If \( A \) is nonsingular, show that the only solution is the trivial one, \( X = O \).

Answers
23. The second row is twice the first row. Hence the two rows are not independent. If one converted \( AX = O \) into two simultaneous equations, the second equation would just be a multiple of the first and could be deleted. Thus we would have one equation and two variables. Thus there is no unique solution.

25. (a) \( A^{-1} = \begin{bmatrix} -2 & 3 \\ 13 & -1 \end{bmatrix} \), (b) \( A^{-1} = \begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix} \)

27. \( A = \begin{bmatrix} 11 & 19 \\ 7 & 0 \end{bmatrix} \)

29. (a) \( B = \begin{bmatrix} 6 \\ 13 \\ 11 \\ 13 \end{bmatrix} \), (b) \( B = \begin{bmatrix} 8 \\ 13 \\ 19 \\ 13 \end{bmatrix} \)

37. If \( A \) is nonsingular, it has an inverse \( A^{-1} \) with \( A^{-1}A = I_n \). Hence \( AB = O_n \) implies \( A^{-1}AB = A^{-1}O_n \) implies \( IB = O_n \) implies \( B = O_n \). Recall \( DO_n = O_n \) by the zero law.

39. If \( A \) is nonsingular, it has an inverse \( A^{-1} \) with \( A^{-1}A = I_n \). If \( AX = O \), then \( A^{-1}AX = A^{-1}O \), and so \( IX = O \), and hence \( X = O \).