1b. Put in reduced row echelon form.

\[
\begin{array}{ccc}
1 & 2 & -3 \\
-1 & 0 & 3 \\
0 & 1 & 2 \\
2 & 3 & 0 \\
\end{array}
\]

3. Let \( A \) be an \( n \times n \) matrix in reduced row echelon form. Prove that if \( A \neq I_n \), then \( A \) has a row consisting entirely of zeros.

In 7abc, solve the linear systems by putting their augmented matrices into reduced row echelon form.

7(a).

\[
\begin{array}{ccc|c}
x & y & z & 0 \\
1 & 1 & 1 & 0 \\
1 & 1 & 0 & 3 \\
0 & 1 & 1 & 1 \\
\end{array}
\]

Solution: \( x = \blank, y = \blank, z = \blank. \)

7(b).

\[
\begin{array}{ccc|c}
x & y & z & 0 \\
1 & 2 & 3 & 0 \\
1 & 1 & 1 & 0 \\
1 & 1 & 2 & 0 \\
\end{array}
\]

Solution: \( x = \blank, y = \blank, z = \blank. \)

7(c).

\[
\begin{array}{ccc|c}
x & y & z & 0 \\
1 & 2 & 3 & 0 \\
1 & 1 & 1 & 0 \\
5 & 7 & 9 & 0 \\
\end{array}
\]

Solution: \( x = \blank, y = \blank, z = \blank. \)

11. Find a \( 2 \times 1 \) column matrix \( X \) other than \( O \) such that \( AX = 4X \)

where \( A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \).

Hint: \( AX = 4X \) iff \( AX - 4X = O \), iff \( (A - 4I)X = O \).

13. Find a \( 3 \times 1 \) column matrix \( X \) other than \( O \) such that \( AX = X \)

where \( A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix} \).

In 15, 17 find all values of \( a \) for which the given system has (a) no solution, (b) a unique solution, (c) (infinitely) many solutions. Write \( \times \) if there are none.

15. \( x + y + z = 2 \)

\( 2x + 3y + 2z = 5 \)

\( 2x + 3y + (a^2 - 1)z = a + 1 \)

(a) no sol. \( a = \blank \), (b) unique sol. \( a \neq \blank \), (c) many sol. \( a = \blank \)

17. \( x + y = 3 \)

\( x + (a^2 - 5)y = a \)

(a) no sol. \( a = \blank \), (b) unique sol. \( a \neq \blank \), (c) many sol. \( a = \blank \)

Answers

1b. \( I_n \)

3. If the lead variable of the last row is not 1, it must be 0 and all entries of the last row are 0.

7. (a) \( x = -1, y = 4, z = 3 \), (b) \( x = y = z = 0 \), (c) \( x = z, y = -2z, z \) arbitrary.

11. Any matrix of the form \( \begin{bmatrix} r \\ r \end{bmatrix} \) with \( r \neq 0 \).

13. Any matrix of the form \( \begin{bmatrix} -\frac{1}{2}r, \frac{1}{2}r, r \end{bmatrix}^T \) with \( r \neq 0 \).

15. (a) \( a = \pm \sqrt{3} \), (b) \( a \neq \pm \sqrt{3} \), (c) no such \( a \).

17. (a) \( a = -3 \), (b) \( a \neq \pm 3 \), (c) \( a = 3 \).