3abc. A is a $3 \times 3$ matrix. For each elementary column operation $e$ below, find an elementary matrix $E$ such that $AE = e(A)$.

(a) $e$ subtracts 4 times the first column from the second column.

(b) $e$ interchanges the second and third columns.

(c) $e$ multiplies the third column by 4.

Hint, the entries in the final answers for 6, 8, 12 are all between -4 and 4. The fractions either have 2 or 4 in the denominator. Hence if you get -5 or 1/3, recheck your work.

9. Invert the following matrices, if possible. Here you need only give the answers. Exactly one matrix has no inverse.

(a) matrix inverse, if any

\[
\begin{bmatrix}
1 & 3 \\
2 & 6
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
1 & 3 \\
-2 & 6
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & 2 & 3 \\
1 & 1 & 2 \\
0 & 1 & 2
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
1 & 2 & 3 \\
1 & 1 & 2 \\
0 & 1 & 1
\end{bmatrix}
\]

Answers

3(a) \[
\begin{bmatrix}
1 & -4 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 4
\end{bmatrix}
\]

9. (a) Singular (b) \[
\begin{bmatrix}
\frac{1}{2} & -\frac{1}{4} \\
\frac{1}{6} & \frac{1}{12}
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
0 & 1 & -1 \\
2 & -2 & -1 \\
-1 & 1 & 1
\end{bmatrix}
\]

(d) Singular

13. Write $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ as a product of elementary matrices. If $e_1, e_2, \ldots, e_n$ is the sequence of elementary row operations you used to find the inverse $B$ of $A$, then $B = E_n \cdots E_2 E_1$. Hence $A = B^{-1} = (E_n \cdots E_2 E_1)^{-1} = D_n \cdots D_2 D_1$, where $D_i$ is the matrix for the inverse of the row operation $e_i$. The inverse operations are:


The product of the corresponding elementary matrices (the matrices which result from applying the operations to $I_2$) is:

$A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$