In 4, 6, the given set and operations do not form a vector space. Cross out the vector-space property that fails. Ignore the * problems, they have been done as examples.

4(2). \( V = \{ (x, y): x \text{ and } y \text{ are real numbers} \}, \quad (x, y) \oplus (r, s) = (x+r, y+s), \quad c \odot (r, s) = (r, cs). \) One property fails.

\[
\begin{align*}
\text{For some } u, v \in V, & \quad u \oplus (v \oplus w) = (u \oplus v) \oplus w, \\
\text{For some } u, v \in V, & \quad (c+d) \odot u = c \odot u \oplus d \odot u.
\end{align*}
\]

For some \( u, v \in V \), \( u \oplus v = u, \) \( c \odot u = u \).

5(3) Prove \( c \odot (d \odot u) = (cd) \odot u \).

6(2). \( V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \leq 0 \right\} \), \( \oplus \) and \( \odot \) are the usual matrix addition and scalar multiplication. One property fails.

\[
\begin{align*}
\text{For some } u, v \in V, & \quad u \oplus (v \oplus w) = (u \oplus v) \oplus w, \\
\text{For some } u, v \in V, & \quad (c+d) \odot u = c \odot u \oplus d \odot u.
\end{align*}
\]

For some \( u, v \in V \), \( u \oplus v = v \oplus u, \) \( c \odot u = u \).

8(7). Prove that the following set and operations form a vector space. No backwards proofs, no missing =’s proofs.

\( V = \{ \text{positive reals} \}, \ u \oplus v = uv \) (ordinary multiplication), \( r \odot u = u^r. \)

1. Prove \( u \oplus v = v \oplus u \).

2. Prove \( u \oplus (v \oplus w) = (u \oplus v) \oplus w \).

3. What number serves as the zero vector \( 0 \) (hint it is not the number 0)? \( \_ \_ \_ \_ \). Prove \( u \oplus 0 = u \).

4. Given \( u \) what is the negative (as a vector) \( -u \)? \( \_ \_ \_ \_ \_ \). Prove \( u \oplus -u = 0 \).

5. Prove \( c \odot (u \oplus v) = c \odot u \oplus c \odot v \).

6. Prove \( (c+d) \odot u = c \odot u \oplus d \odot u \).

7. Prove \( c \odot (d \odot u) = (cd) \odot u \).

8. Prove \( 1 \odot u = u \).

In 10, 12, cross out the vector-space properties that fail.

10(7). Prove the following set and operations are a vector space.

\( V = \{ 0 \}, \ 0 \oplus 0 = 0, \ r \odot 0 = 0 \).

1. Prove \( u \oplus v = v \oplus u \).

2. Prove \( u \odot (v \oplus w) = (u \odot v) \oplus w \).

\( u \odot (v \oplus w) = 0 \oplus (0 \odot 0) = 0 \odot 0 = 0 \)

\( (u \odot v) \oplus w = (0 \odot 0) \oplus 0 \odot 0 = 0 \)

(3) What number serves as the zero vector \( 0 \)? \( \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \). Prove \( u \oplus 0 = u \).

4. Given \( u \) what is the negative (as a vector) \( -u \)? \( \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \). Prove \( u \oplus -u = 0 \).

5. Prove \( c \odot (u \oplus v) = c \odot u \oplus c \odot v \).

6. Prove \( (c+d) \odot u = c \odot u \oplus d \odot u \).

7. Prove \( c \odot (d \odot u) = (cd) \odot u \).

8. Prove \( 1 \odot u = u \).

† Note: if “For some \( 0, u \oplus 0 = u \)” fails, so does “\( u \oplus -u = 0 \) for some -u”.

Exam 1 Wed. This is due Friday. Hw 102: 4, 6, 10, 12, 14. Recommended 102: 3, 5, 7, 9, 13. Answers 2.2, 539.