Math 311    Hw 9   Recommended problems, don’t turn this in.

Exam 1 Wed. This is due Friday. Hw 102: 4, 6, 8, 10, 12, 14.   Recommended 102: 3, 5, 7, 9, 13.  Answers 2.2, 539.

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In 3, 5, 7 the given set and operations are not a vector space. Cross out the vector-space property that fails.

3. \( V = \{ x: x > 0 \} \), \( x \oplus y = x + y \), \( c \otimes x = cx \).
   1. \( u \oplus v = \ominus u \)
   2. \( u \ominus (v \oplus w) = (u \ominus v) \ominus w \)
   3. For some \( u \), \( u \ominus 0 = u \)
   4. \( u \ominus u = 0 \) for some \(-u\)
   5. \( v = v \)

5. \( V = \{(x,y,z): x,y,z \text{ are real numbers}, (x,y,z) \ominus (r,s,t) = (x+r, y+s, z+t)\} \), \( c \ominus (r,s,t) = (c \ominus r, c \ominus s, c \ominus t) \).
   1. \( u \ominus v = \ominus u \)
   2. \( u \ominus (v \ominus w) = (u \ominus v) \ominus w \)
   3. For some \( u \), \( u \ominus 0 = u \)
   4. \( u \ominus u = 0 \) for some \(-u\)

7. \( V = \{(x,y): x,y \text{ are real numbers}, (x,y) \ominus (r,s) = (x+r, y+s)\} \), \( c \ominus (r,s) = (c \ominus r, c \ominus s) \).
   1. \( u \ominus v = \ominus u \)
   2. \( u \ominus (v \ominus w) = (u \ominus v) \ominus w \)
   3. For some \( u \), \( u \ominus 0 = u \)
   4. \( u \ominus u = 0 \) for some \(-u\)

9. Prove that the following set and operations form a vector space. Answer is at the bottom of each problem.
   \( V = \{\text{real-valued continuous functions}, f \oplus g = f+g \text{ where } (f+g)(t) = f(t)+g(t), \quad r \otimes f = rf \text{ where } (rf)(t) = r(f(t))\} \).

1. Prove \( f \oplus g = g \oplus f \).

   \((f \oplus g)(t) = f(t)+g(t) = g(t)+f(t) = (g \oplus f)(t)\)

2. Prove \( f \oplus (g \oplus w) = (f \oplus g) \oplus w \).

   \((f \oplus (g \oplus w))(t) = f(t)+(g \oplus w)(t) = f(t)+(g(t)+w(t)) = f(t)+g(t)+w(t) = (f \oplus g)(t)+w(t) = ((f \oplus g) \oplus w)(t)\)

3. What serves as the zero vector \( 0 \)?  ______ .
   Prove \( f \oplus 0 = f \).

   \((f \oplus 0)(t) = f(t)+0(t) = f(t) \)

In 12, 14, cross out the vector-space properties that fail.

13. \( V = (-\infty, \infty) = \text{all reals} \)
    \( u \oplus v = uv \), \( c \otimes u = cu \).

   1. \( u \oplus v = v \oplus u \)
   2. \( u \oplus (v \oplus w) = (u \oplus v) \oplus w \)
   3. For some \( u \), \( u \oplus 0 = u \)
   4. \( u \oplus u = 0 \) for some \(-u\)

Answers

3. Properties 3, 4, 5, 6, 7.
5. Properties 5, 6, 8.
7. Property 8.
13. Properties 4, 5, 6, 7, 8.