**Math 311  Lecture 13**

**THEOREM.** Suppose U and W are subsets of a vector space V, and suppose W independent and W spans V, hence by (a) \( u \leq w \).

**PROOF.** (a) By the second theorem, W can be extended to a basis. By hypothesis, no strictly larger set is independent. Hence it must be a basis.

(b) By the first theorem, a subset of W is a basis. By hypothesis, no strictly smaller set spans V. Hence W is a basis.

**COROLLARY.** Suppose W is a subset of a vector space V, W has w elements and V has dimension n.

(a) W independent implies \( w \leq n \); \( w > n \) implies W is not span.

(b) W spans V implies \( w \geq n \); \( w < n \) implies W doesn’t span V.

(c) W independent and \( w = n \) implies W is a basis.

(d) W spans V and \( w = n \) implies W is a basis.

**PROOF.** (a) and (b) follow the theorem above. (c) and (d) follow from the corollary above.

**Definition.** The dimension \( n \) of a vector space V is the number of elements in any basis of V. If \( V = \{0\} \), the dimension is 0.

**Corollary.** Suppose W is a subset of a vector space V, W has w elements and V has dimension n.

(a) W independent implies \( w \leq n \); \( w > n \) implies W is not span.

(b) W spans V implies \( w \geq n \); \( w < n \) implies W doesn’t span V.

(c) W independent and \( w = n \) implies W is a basis.

(d) W spans V and \( w = n \) implies W is a basis.