If the vector space has cols / rows, your basis must have cols / rows, respectively. Write vectors in factored form.

1. \( W = \text{span} \left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \right\} \). Find a basis for \( W^\perp \).

5. \( V = P_3 = \) the vector space of polynomials of degree 3 or less. The inner product is defined as before by \( f \cdot g = \int_0^1 f(t)g(t)dt \). \( W = \) the space spanned by \( \{ t - 1, t^2 \} \). Find a basis for \( W^\perp \).

In 15 and 21, \( W \) is the subspace of \( \mathbb{R}^3 \) with orthonormal basis \( \{ w_1, w_2 \} \) where \( w_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \), \( w_2 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \).

15. Write the vector \( v = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \) as \( w + u \) with \( w \in W \) and \( u \in W^\perp \).

21. Let \( v = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \). Find the distance between \( v \) and \( W \).

Answers

1. \( \{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \} \)

5. \( \{ \frac{45}{14}t^3 - \frac{55}{12}t^2 + t, \frac{130}{7}t^3 - \frac{130}{7}t^2 + 1 \} \)

15. \( w = \frac{1}{5} \begin{bmatrix} -1 \\ 10 \\ -2 \end{bmatrix} \), \( u = \frac{1}{5} \begin{bmatrix} 6 \\ 0 \\ -3 \end{bmatrix} \)

19. \( \frac{3}{5} \sqrt{5} \)