2(2). Find the inverses of the following orthogonal matrices (if you can’t immediately write the inverse, review the lecture).

(a) \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{bmatrix}
\]
(b) \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{bmatrix}
\]

Hint: all eigenvalues & all entries in the D’s are digits in \{-3,-2,-1,0,1,2,3\}.

14(3). Suppose \(B = PAP^{-1}\) and that \(\lambda\) is an eigenvalue of \(A\) with eigenvector \(v\). Prove that \(\lambda\) is an eigenvalue for \(B\) with eigenvector \(Pv\).

In 16, 18, 20, find a diagonal matrix \(D\) and an orthogonal matrix \(P\) such that \(A = PDP^{-1}\).

Eigenvalues of degree \(k\) must have \(k\) eigenvectors and must occur \(k\) times on \(D\)’s diagonal. On \(D\)’s diagonal, list the positive entries first, then the negatives, then the 0’s. The \(D\)’s have single-digit entries. For the entries in the \(P\)’s, there are 15 0’s, 3 1’s, 10 with \(\sqrt{2}\), 3 with \(\sqrt{3}\), 3 with \(\sqrt{6}\). Please check that the columns of the \(P\)’s are orthogonal.

16(3). \(A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\)

18(3). \(A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\)

20(3). \(A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}\)

In 22, 24, 26, 28, find the diagonal matrix \(D\) for \(A\). You don’t have to find the \(P\), to find \(D\), you need only the eigenvalues, not the eigenvectors.

An eigenvalue of degree \(k\), must occur \(k\) times along \(D\)’s diagonal. The entries of each \(D\) are positive/negative digits in \{-4,-3,-2,-1,0,1,2,3,4,5\}. On \(D\)’s diagonal, list the positive entries first, then negatives, then the 0’s.

22(1). \(A = \begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}\)

24(1). \(A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & -2 & 3 \end{bmatrix}\)

26(1). \(A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\)

28(1). \(A = \begin{bmatrix} -3 & 0 & -1 \\ 0 & -2 & 0 \\ -1 & 0 & -3 \end{bmatrix}\)