Math 311  Review 2

Exam office hours: Tuesday, 10:30-12:30.
Graded homework is placed in the bin on my office door (PSB 318) and may be picked up after 2:00.
No calculators for this or subsequent exams.


Definitions
A vector space, closed under addition and scalar multiplication. subspace. The null space and the nullity of A.
The span of a set of vectors, span S. S spans V. Vectors v₁, v₂, ..., vₙ are linearly dependent or independent. A basis for a vector space. The standard basis. The dimension of a vector space V.
[v]ₜ, the coordinate vector of v w.r.t. W. Pₜ⁻¹ the transition matrix from T to W.
The row space (column space) of a matrix and its row rank (column rank).

Theorems
Theorem. The 0 is unique. So is -u.
Lemma (Cancellation). u@v = u@v’ implies v = v’.
Theorem. For any vector space V, any uєV, and scalar c:
(a) 0@u = 0.
(b) c@0 = 0.
(c) If c@u = 0 then c = 0 or u = 0.
(d) -(1)@u = -u
Lemma (statement only). Given vectors v₁, v₂, ..., vₙ written as column vectors of Rⁿ, if A = [v₁, v₂, ..., vₙ] is the matrix whose columns are v₁, v₂, ..., vₙ then v₁, v₂, ..., vₙ span Rⁿ iff the rref of A does not have a row of 0’s.
Lemma (statement only). Given vectors v₁, v₂, ..., vₙ written as column vectors of Rᵐ, if A = [v₁, v₂, ..., vₙ] is the matrix whose columns are v₁, v₂, ..., vₙ then v₁, v₂, ..., vₙ are independent iff the rref of A has n nonzero rows.
Theorem. If v₁, v₂, ..., vₙ are a basis for V, then every vector of V can be written in one and only one way as a linear combination of v₁, v₂, ..., vₙ.
Corollary (statement only). Suppose V is a vector space.
(a) If W is a maximal set of independent vectors (i.e., it can’t be extended to a larger set of independent vectors) then W is a basis.
(b) If W is a minimal set of vectors which span V (i.e., no proper subset of W spans V) then W is a basis.
Theorem (statement only). Suppose U and W are subsets of a vector space V and suppose U has u elements and W has w elements.
(a) If U is independent and W spans V, then u ≤ w.
(b) If U and W are both bases, then u = w.
Corollary. Suppose W is a subset of a vector space V, W has w elements and V has dimension n.
(a) W independent implies w ≤ n; w > n implies W is dependent.
(b) W spans V ⇒ w ≥ n; w < n ⇒ W doesn’t span V.
(c) W independent and w = n implies W is a basis.
(d) W spans V and w = n implies W is a basis.
Lemma. (a) Pₜ⁻¹[v]₁ = [v]ₜ for any vector v є V.
(b) Pₜ⁻¹Pₖ⁻¹ = Pₜ⁻¹.
(c) Pₜ⁻¹ = (Pₜ⁻¹)⁻¹
Lemma. If v є Rⁿ and U is the standard basis and W and T are any other bases:
(a) [v]ₜ = v
(b) Pₜ⁻¹ = (w₁, w₂, w₃, ...)
(c) Pₜ⁻¹ = (Pₜ⁻¹)⁻¹
(d) [v]ₜ = Pₜ⁻¹[v]
(e) Pₜ⁻¹ = Pₜ⁻¹Pₜ⁻¹

Theorem (statement only). The rows of a matrix’s rref matrix are a basis for the row space. The row rank of a matrix = the number of nonzero rows of its rref matrix.
Theorem. The row rank equals the column rank of a matrix.

Main Techniques Be able to
Identify vector spaces and subspaces.
Determine if a set of vectors span a space, if they are independent, if they are a basis.
Write a vector as a linear combination of other vectors or determine that it is not.
Write parametric equations for a given line.
Find bases for given subspaces or for a null space.
Given a set of vectors: Column method - find a subset which is a basis for the space spanned by the vectors.
Row method - find a simple basis for the spanned space.
Be able to find transition matrices and given [v]ₜ be able to find [v], given bases S and T.
Find λ such that AX = λX has a nontrivial solution for a given A.

Suggested Exercises. All homework exercises plus the recommended exercises.

Page Problem
102: 3, 5, 7, 9, 13.
111: 1abcd, 3abc, 5ab, 7abc, 9abc, 11ab, 23abcd, 31ab.
122: 1abcd, 3abd, 5, 7, 11abc. 13abc.
137: 1abcd, 3abc, 5bc, 7ab, 9a, 11, 17, 19ac, 21, 29.
147: 3, 5, 13, 15, 17, 19.
161: 1, 3, 7, 9, 13, 19, 23, 25.
169: 1, 5, 19ab, 29ab.

Hw 15 Answers