Suppose a nickel and a dime are tossed. There are 4 outcomes: \{HH, HT, TH, TT\} where HT means the nickel was heads and the dime was tails. The first letter represents the nickel; the second is the dime.

The probability of H on the nickel is \(\frac{1}{2}\), the probability of T on the dime is also \(\frac{1}{2}\), hence the probability of HT is \(\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\). Likewise HH, TH, TT have probability \(\frac{1}{4}\).

Consider the following events:

- **A** = head on nickel
- **B** = tail on nickel
- **C** = head on dime
- **D** = tail on dime

Thus **A**, “head on nickel”, is the subset \{HH, HT\} of outcomes for which the nickel was heads. Likewise **B** = “tail on nickel” = \{TH, TT\}, **C** = “head on dime” = \{HH, TH\}.

Clearly \(P(A) = P(B) = P(C) = P(D) = \frac{1}{2}\).

Using the definition gives the same answer:

\[
P(A) = \text{sum of probabilities of outcomes in } A = P(HH) + P(HT) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.
\]

**Mutually exclusive** means “disjoint” or “inconsistent” or “can’t both happen”. **Independent** means “knowing that one event has or has not happened doesn’t provide any information about the other.

Are **A** (head on nickel) and **B** (tail on nickel) mutually exclusive? Independent?

They are mutually exclusive, you can’t get a head on the nickel and a tail.

They are not independent. If you know that A is true, then you know something about B, i.e., that B is false. In fact mutually exclusive events are never independent.

Are **A** (head on nickel) and **C** (head on dime) mutually exclusive? Independent?

They are not mutually exclusive. HH makes both A and C true, so A and C can both happen.

They are independent, know what happened with the nickel does not provide any information about what happened with the dime.

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4.65 A sample is selected from one of two populations, \(S_1\) and \(S_2\), with probabilities \(P(S_1) = .7\) and \(P(S_2) = .3\). If the sample has been selected from \(S_1\), the probability of observing event **A** is \(P(A | S_1) = .2\). Similarly, if the sample has been selected from \(S_2\), the probability of observing **A** is \(P(A | S_2) = .3\).

(a) If a sample is randomly selected from one of the two populations, what is the probability that event **A** occurs?

Just add up the pieces in **A**.

\[
(\cdot .7)(.2) + (.3)(.3) = .23
\]

(b) If the sample is randomly selected and event **A** is observed, what is the probability that the sample was selected from population \(S_1\)?

Saying “**A** is observed” is another was of saying the “**A** is given”. Hence the probability we want is the conditional probability of \(S_1\) given \(A\),

\[
P(S_1 | A) = \frac{P(S_1 \cap A)}{P(A)} = \frac{(\cdot .7)(.2)}{.23} = .61
\]

(c) If the sample is randomly selected and event **A** is observed, what is the probability that the sample was selected from population \(S_2\)?

This is the conditional probability of \(S_2\) given \(A\),

\[
P(S_2 | A) = \frac{P(S_2 \cap A)}{P(A)} = \frac{(\cdot .3)(.3)}{.23} = .39
\]

A red and a green die are rolled. The number that can come up on either die is 1, 2, 3, 4, 5, or 6. You get a “double” if the red and green numbers are the same. If you get a double, you win the total amount on the red and green die. Otherwise you win nothing. What are your expected winnings?

\[
S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, ..., 66\} where 12 means 1 on the red and 2 on the green.
\]

\[
\text{Expected winning} = (\text{amount for 11})(\text{probability of 11})+ (\text{amount for 22})(\text{probability of 22})+ \ldots + (\text{amount for 66})(\text{probability of 66}) + \text{(amount for other outcomes)}\times (\text{probability of other outcomes})
\]

\[
= (2)(1/36)+4(1/36)+6(1/36)+8(1/36)+10(1/36)+12(1/36)+0(30/36)
\]

\[
= (2+4+6+8+10+12)(1/36) = 42/36 = $1.67
\]

.5 of all students are male, .4 of male students are in science, .2 of female students are. What percentage of science students are male?

\[
\text{Percentage} = \frac{.2/(.2+1.1)}{.2/3} = 2/3 = .67 = 67\% 
\]