5.41 Let $x$ be the number of successes observed in a sample of $n = 5$ items selected from $N = 10$. Suppose that, of the $N = 10$ items, 6 are considered "successes". Find the probability of
(a) no successes $P(x=0) = 0$
This is impossible and hence has probability 0.
(b) at least two successes $P(x\geq 2) = 1 - P(x=0) - P(x=1) = 1 - 0 - \frac{\binom{M}{k} \binom{N-M}{n-k}}{\binom{N}{n}}$
\[= 1 - \frac{6 \cdot \binom{4}{1}}{\binom{10}{5}} = 1 - \frac{6 \cdot \frac{6!}{1!5!}}{\frac{10!}{5!5!}} = 1 - .0238 = .976 = .98\]
(c) exactly two successes
\[\frac{\binom{M}{k} \binom{N-M}{n-k}}{\binom{N}{n}} = \frac{\binom{2}{2} \binom{3}{3}}{\binom{5}{5}} = .238 = .24\]

5.45 A company has 5 applicants for 2 positions: 2 women and 3 men. Suppose that the 5 applicants are equally qualified and that no preference is given for choosing either gender. Let $x$ equal the number of women chosen to fill the 2 positions.

Since $x$ counts "success", success is being a woman.
\[\because \, M = \text{number of successes} = 2.\]
\[N-M = \text{number of failures} = 3.\]
\[N = \text{the total number of applicants} = 5.\]
The sample = the subset of the the 5 applicants who are chosen for the two positions.
\[\therefore \, n = 2.\]
(a) Write a formula for $p(x)$, the probability of exactly $x$ successes.
\[\frac{\binom{3}{x} \binom{2}{2-x}}{\binom{5}{2}}\]
(b) What are the mean and variance of this distribution?
\[\mu = \frac{nM}{N} = \frac{2}{5} = \frac{4}{5} = .8\]
\[\sigma = \sqrt{\frac{nM-NM-n}{N(N-1)}} = \sqrt{\frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}{5 \cdot 4 \cdot 4} = \sqrt{\frac{9}{25} = .6}}\]