11.3. You have 6 treatment populations based on independent random samples (hence they have a common std. dev.). Each has 10 observations. Suppose total SS = 21.4 and SSE = 16.2. Suppose \( \bar{x}_1 = 3.07 \) and \( \bar{x}_2 = 2.52 \).

(a) Construct the (two-sided) 95% confidence interval for \( \mu_1 \) around \( \bar{x}_1 \).

95% confidence interval =

(b) Construct the 95% confidence interval for \( \mu_1 - \mu_2 \) around the difference \( \bar{x}_1 - \bar{x}_2 \).

95% confidence interval =

11.5. You have 4 populations based on independent random samples. Each has 6 observations. Suppose total SS = 473.2, SST = 339.8

(a) How many degrees of freedom are associated with the F statistic for testing \( H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \)?

\[ df_1 = \text{m} \]

(b) Find the acceptance region for \( H_0 \) with \( \alpha = 5\% \).

(F\text{\textunderscore }0, 3.10)

(c) Should we accept the null hypothesis?

(d) Find the p-value interval for this test.

Answers

11.3. You have 6 populations based on independent random samples. Each has 10 observations. Suppose total SS = 21.4 and SSE = 16.2. Suppose \( \bar{x}_1 = 3.07 \) and \( \bar{x}_2 = 2.52 \).

Since the populations all have the same variance, the best estimate of the population std. dev. is not the individual samples std. devs. \( s_i \) but the pooled std. dev. \( s \).

\[ df_E = n-k = 60-6 = 54 \]

\[ s = \sqrt{MSE} = \sqrt{SSE/df_E} = \sqrt{16.2/54} = .5477 \]

\[ SE_{x_1} = \frac{s}{\sqrt{n}} = \frac{.5477}{\sqrt{10}} = .1732 \]

\[ SE_{x_1-x_2} = s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = .5447 \sqrt{\frac{2}{10}} = .2449 \]

(a) Construct the (two-sided) 95% confidence interval around \( \bar{x}_1 \) for \( \mu_1 \).

95% confidence interval =

\[ 3.07 \pm 1.96 \times .1732 = [2.731, 3.409] \]

(b) Construct the 95% confidence interval for \( \mu_1 - \mu_2 \) around the difference \( \bar{x}_1 - \bar{x}_2 \).

95% confidence interval =

11.5. You have 4 populations based on independent random samples. Each has 6 observations. Suppose total SS = 473.2, SST = 339.8

\[ SSE = totalSS - SST = 473.2 - 339.8 = 133.4 \]

\[ df_E = n-k = 24-4 = 20, \quad df_T = k-1 = 3 \]

\[ MSE = SSE/df_E = 133.4/20 = 6.67 \]

\[ MST = SST/df_T = 339.8/3 = 113.2667 \]

\[ F = MST/MSE = 113.2667/6.67 = 16.9815 \]

(a) How many degrees of freedom are associated with the F statistic for testing \( H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \)?

\[ df_1 = df_T = 4-1 = 3, \quad df_2 = df_E = 24-4 = 20 \]

(b) Find the acceptance region for \( H_0 \) with \( \alpha = 5\% \).

\[ F \in [0, 3.10] \]

(c) Should we accept the null hypothesis?

No, \( F = 16.98 \not\in [0, 3.10] \)

(d) Find the p-value interval for this test.

\[ p-value \in [0, .005] \]