Math 414 Lecture 8

Solve by geometry and then by the simplex algorithm.

Max \( z = x + y \)
with \( r: 2x + y \leq 6 \)
\( s: x + 2y \leq 6, \quad x, y \geq 0. \)

\[
\begin{align*}
(0,0) & \quad (0,3) \quad (2,2) \quad \text{(2,2)} \quad \text{(3,0)} \quad 6
\end{align*}
\]

Now check the answer using LPSolve.

**Definition.** A basic variable is **degenerate** iff its value is 0. A basic solution is **degenerate** iff at least one of its basic variables is degenerate. A degenerate solution has a 0 in the constant column. The objective value at the bottom is not considered part of the constant column.

A degenerate solution arises whenever there are two (or more) minimal constant/(positive-coefficient) ratios. This means that two basic variables go to 0 simultaneously. Since only one variable can be changed at a time, one is chosen to be the departing variable and becomes a parameter. The other remains a basic variable but since it is 0, it is degenerate. It takes an additional step to make the degenerate basic a parameter.

**Cycling** can (but usually doesn’t) occur in this situation. Suppose \( x \) and \( y \) are basic variables which both go to 0. Suppose we choose \( x \) to be the departing variable. Thus it becomes a parameter and could become the next entering basic. If \( x \) and \( y \) both go to zero again, don’t choose \( x \) as the departing variable again or you will get into a repeating cycle.

Recall from Lecture 7.

In row \( r \), the \( \theta \)-ratio \( \theta_r = \text{constant/(positive-coefficient)} \) is the amount the entering variable can increase before the basic variable \( r \) goes to 0.

If the ratio is 0, then the entering variable can’t increase at all. Thus the set of basic variables changes but the basic values are the same. In the picture below, the two basic solutions have different basic variables (the basics are underlined) but the solutions are the same. Thus moving from one to the other leaves one at the same extreme point: no movement takes place, no values change. The intermediate change of variables has to be made in order to make the two changes of variables needed to produce one change in location and value.
**Problem:**

Maximize \( z = 2y + x \)

Subject to:

- \(-x + y \leq 3\)
- \(y \leq 3\)
- \(x \leq 3\)
- \(x, y \geq 0\)

**Optimal solution:**

Maximize \( z = \), when \( x = \), \( y = \)

We didn’t list \( r = 3 \) as part of the final answer, it is a slack variable, not one of the original variables.