1(4). Prove that the dual of the dual of a primal problem is equivalent to the primal problem for the example below.
Find the dual and then find the dual of the dual.
Here $a, b, c, d, e, f, p, q$ are constants.

<table>
<thead>
<tr>
<th>PRIMAL</th>
<th>DUAL</th>
<th>DUAL OF DUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min $z = px + qy$</td>
<td>with $r$: $ax + by \leq e$</td>
<td>with $s$: $cx + dy \geq f$</td>
</tr>
<tr>
<td>$x, y \geq 0$</td>
<td></td>
<td>$x, y \geq 0$</td>
</tr>
</tbody>
</table>

2(6) Solve the primal, canonical and dual problem from the geometric representation of the primal problem.

<table>
<thead>
<tr>
<th>PRIMAL</th>
<th>CANONICAL</th>
<th>DUAL</th>
<th>OPTIMAL SOLUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max $z = y$</td>
<td>max $z = y$</td>
<td>min $z =$</td>
<td>$z =$</td>
</tr>
<tr>
<td>$r$: $-x + y \leq 2$</td>
<td>$r$:</td>
<td>$x$:</td>
<td>when</td>
</tr>
<tr>
<td>$s$: $x + y \leq 6$</td>
<td>$s$:</td>
<td>$y$:</td>
<td>$y =$</td>
</tr>
<tr>
<td>$t$: $x - y \leq 2$</td>
<td>$t$:</td>
<td></td>
<td>slacks</td>
</tr>
<tr>
<td>$x, y \geq 0$</td>
<td></td>
<td></td>
<td>duals</td>
</tr>
</tbody>
</table>

Check: $z + x + y = 10$, total slacks = 4, total duals = 1

Draw in each of the three shifted lines $r+1$, $s+1$, $t+1$ which are needed to calculate the dual variables $r$, $s$, and $t$ via the Marginal Value Theorem.

This is almost identical to Prob. 2 of Hw 10 except that you have to solve it geometrically.
1(4) You make lamps. You wish to maximize your profit.
Profit/lamp = $30. \quad x = \# \text{ of lamps you make per day.}
\[ z = \text{total profit per day.} \quad w = \text{the dual variable.} \]
Max \# \text{ of lamps per day you can make} = 6.
A customer wishes to employ you to make 6 lamps a day.

\[
\begin{array}{c}
\text{PRIMAL} & \text{DUAL} \quad \text{(complete the dual problem)} \\
\text{max } z = 30x & \text{min } z' = \\
\text{with } w: x \leq 6 & \text{with } x: \quad \quad \\
x \geq 0 & \quad \quad \\
\end{array}
\]
\[
\text{max } z = 180 \text{ when } x = 6, \quad \text{min } z' = \quad \text{when } w = \quad .
\]

Give an economic interpretation of \( w \) and \( z' \) in terms of wages (circle the best interpretation or fill in “other”).

\[
\begin{array}{c}
w = \text{wage for one day’s work} \\
w = \text{wage charged for making one lamp} \\
\text{hourly wage} \\
\text{Other } \quad \quad \\
\end{array}
\]
\[
\begin{array}{c}
z' = \text{total wage} \\
\text{one day’s wage} \\
\text{one day’s profit} \\
\text{profit per lamp} \\
\text{Other } \quad \quad \\
\end{array}
\]

Give a marginal value interpretation of \( w \).

\[
w = \text{the amount of increase of } \quad \text{when } \quad \quad \\
\quad \quad \\
\]

Hint: check that the units of your economic interpretations are correct. Hence if the Marginal Value Theorem says \( w = \Delta z/\Delta b \) and \( z \) is in $ and \( b \) is in kg., then \( w \) should be in $/kg.

2(9) A coffee packer blends Brazilian coffee and Colombian coffee to prepare two products: Super and Deluxe brands.
- Each kilogram of Super coffee contains 0.5 kilogram of Brazilian coffee and 0.5 kilogram of Colombian coffee.
- Each kilogram of Deluxe coffee contains 0.25 kilogram of Brazilian coffee and 0.75 kilogram of Colombian coffee.
- The packer has 120 kilograms of Brazilian coffee and 160 kilograms of Colombian coffee on hand.
- The profit on each kilogram of Super coffee is 20 cents; the profit on each kilogram of Deluxe coffee is 30 cents.
- How many kilograms of each type of coffee should be blended to maximize profit?

Let \( s, d \) be the number kilograms of Super and Deluxe coffee; let \( P = \text{profit} \).
A coffee buyer wishes to buy all the packer’s coffee for \( T \) cents.
Let \( b, c \) be the slack variables in the primal problem.
Thus \( b = 120 - (.5s + .25d) \).

\[
\begin{array}{c}
\text{State the dual problem using variables } b \text{ and } c. \\
\text{PRIMAL} & \text{DUAL} \\
\text{max } P = 20s + 30d & \text{min } T = \\
\text{with } b: .5s + .25d \leq 120 & \text{with } s: \quad \quad \\
\text{c: } .5s + .75d \leq 160 & \quad \quad d: \quad \quad \\
\text{s, d} \geq 0 & \quad \quad b, c \geq 0 \\
\end{array}
\]

Find the primal slack variables. \( b = \quad \), \( c = \quad \).
Hint: of \( b \) and \( c \), one is a digit, one is a fraction.

Solve the dual problem.
\[
\text{min } T = \quad \text{when } b = \quad \text{, } c = \quad \text{.}
\]

Give an economic interpretation of the dual variable \( b \).
\( b = \quad \)

In the primal problem, if 160 is increased to 161,
\( P \) becomes \quad \quad .
In the primal problem, if 120 is reduced to 119,
\( P \) becomes \quad \quad .

The sum of the last two answers is 12840.