1 Primal problem. The same the region as in Hw 15.
max \( z = x + y \)
with
\[
\begin{align*}
  r: & \quad y \leq 2 \\
  s: & \quad x + y \leq 3 \\
  t: & \quad x \leq 2 \\
  x, y & \geq 0
\end{align*}
\]
Initial matrix

\[
\begin{array}{cccccc|c}
 x & y & r & s & t & b \\
\hline
 r & 0 & 1 & 1 & 0 & 0 & 2 \\
 s & 1 & 1 & 0 & 1 & 0 & 3 \\
 t & 1 & -1 & 0 & 0 & 1 & 2 \\
 & -1 & -1 & 0 & 0 & 0 & 0 \\
\end{array}
\]

(a) Shade the submatrices \( T \); complete the final tableau

(b) (c) (d)

\[
\begin{array}{cccccc|c}
 x & y & r & s & t & B \\
\hline
 0 & 1 & 1 & 0 & 0 & 2 \\
 1 & 0 & -1 & 1 & 0 & 3-p \\
 0 & 0 & 1 & -1 & 1 & p-1 \\
 & & & & & 3 \\
\end{array}
\]

(b) Suppose \( b_r = 2 \) is replaced by the variable \( p \). See columns (b). Write the optimal solution in terms of \( p \) and find the interval for which this solution is feasible.

max \( z = \) \quad at \( x = \) \quad, \( y = \) \quad
for \( p \in \) \quad. \quad Sum of integer endpoints = 4.

(c) Suppose \( b_s = 3 \) is replaced by the variable \( p \). Fill in columns (c). Write the optimal solution in terms of \( p \) and find the interval for which this solution is feasible.

max \( z = \) \quad at \( x = \) \quad, \( y = \) \quad
for \( p \in \) \quad. \quad Sum of integer endpoints = 6.

(d) Suppose \( b_t = 2 \) is replaced by the variable \( p \). Fill in columns (d). Write the optimal solution in terms of \( p \) and find the interval for which this solution is feasible.

max \( z = \) \quad at \( x = \) \quad, \( y = \) \quad
for \( p \in \) \quad. \quad Sum of integer endpoints = 1.

To find constant column sensitivity with LPSolve --

Enter and run the primal problem.
Click “Result/Sensitivity/Duals”
If the range of the constant \( b_r \) for constraint \( r \) is \( \{a, b\} \),
then in the row for constraint \( r \),
the “value” column gives the dual variable \( r \),
the “from” column gives \( a \) and
the “till” column gives \( b \).

value from till
\[
\begin{array}{rr}
  r & a & b \\
\end{array}
\]

Warning: LPSolve's solution for the previous problem is not the one given, changing max \( z = x + 1.1y \) tilts the objective in favor of the one given. Secondly, if the solution is not on a constraint bounday, LPSolve wrongly lists the interval as being all reals (-inf, inf).

To find objective coefficient sensitivity with LPSolve --

Enter and run the primal problem.
Click “Result/Sensitivity/Objective”
If the range of the objective coefficient \( c_x \) is \( \{a, b\} \),
then in the row for coefficient \( x \),
the “from” column gives \( a \) and
the “till” column gives \( b \).
Ignore “from value” and “till value” columns.

2 Solve using LPSolve.
max \( z = y \) with
\[
\begin{align*}
  r: & \quad -x + y \leq 2 \\
  s: & \quad x + y \leq 6 \\
  t: & \quad x - y \leq 2 \\
  x, y & \geq 0
\end{align*}
\]
Optimal solution. max \( z=4 \) when \( x=2, y=4 \).

Find the intervals for which this solution is feasible.
(a) \( b_r = 2 \) is replaced by the variable \( p \). \( p \in \)
Sum of integer endpoints = 4.
(b) \( b_s = 6 \) is replaced by \( p \). \( p \in \)
Sum of integer endpoints = 2.

Find the intervals for which this solution is optimal.
(c) \( c_x = 0 \) is replaced by the variable \( p \). \( p \in \)
Sum of integer endpoints = 0.
(d) \( c_y = 1 \) is replaced by \( p \). \( p \in \)
Primal problem for problems 3, 4, 5.

\[
\text{max } z = \ldots
\]

with

\[
\begin{align*}
  r: & -x + y \leq 1 \\
  s: & x \leq 1 \\
  & x, y \geq 0
\end{align*}
\]

3(__/3) Sketch the region of feasible solutions. For each extreme, draw lines to indicate the pie-shaped sector of objective coefficient vectors for which it is optimal. One segment has been partially done for you.

For problems 4 and 5, suppose the objective function is

\[
\text{max } z = x + y.
\]

Then the final tableau is

\[
\begin{array}{cccccc}
1 & 1 & 0 & 0 & 0 & 0 \\
\hline
1 & y & 1 & 1 & 1 & 2 \\
1 & x & 1 & 0 & 0 & 1 \\
1 & z & 0 & 0 & 1 & 2 \\
\hline
\end{array}
\]

with solution max \( z = 3 \) at \( x=1, y=2 \).

4(__/3) What is the range for \( c_x = 1 \leftrightarrow p \) such that \( x=1, y=2 \) remains optimal? Fill in the blanks, then give the answer.

\[
\begin{array}{cccccc}
1 & 1 & 0 & 0 & 0 & 0 \\
\hline
1 & y & 1 & 1 & 1 & 2 \\
1 & x & 1 & 0 & 0 & 1 \\
1 & z & 0 & 0 & 1 & 2 \\
\hline
\end{array}
\]

The solution is optimal

\[
\text{iff}
\]

\[
\text{iff}
\]

Answer. The range is \( c_x = p \in \)

5(__/3) What is the range for \( c_y = 1 \leftrightarrow p \) such that \( x=1, y=2 \) remains optimal? Fill in the blanks, then give the answer.

\[
\begin{array}{cccccc}
1 & 1 & 0 & 0 & 0 & 0 \\
\hline
1 & y & 1 & 1 & 1 & 2 \\
1 & x & 1 & 0 & 0 & 1 \\
1 & z & 0 & 0 & 1 & 2 \\
\hline
\end{array}
\]

The solution is optimal

\[
\text{iff}
\]

\[
\text{iff}
\]

Answer. The range is \( c_y = p \in \)