Math 414   Practice Exam 1   Hw  ___/55
Sit two apart during the exam. Develop enough proficiency to do this in 75 minutes. Homework credit given for problems 5-15 if turned in on exam day. Bring a laptop and scratch paper. Generally one or more problems will be recommended problems, do these ahead of time.

Definitions and Theorems
1(0) Definitions. For \( x, x_1, x_2, \ldots, x_n \in \mathbb{R}^n \) and \( S \subseteq \mathbb{R}^n \), for \( A \) a matrix,
(a) \( x \) is an extreme point of \( S \) iff
(b) \( x \) is a convex combination of \( x_1, x_2, \ldots, x_n \) iff
(c) \( A \) is in reduced row echelon form iff
(d) A set of basic variables is a set of variables s.t. ...

2(0) For canonical systems, characterize extreme points \( p \) in terms of the number zeros.

3(0) Theorem. If \( S = \{v_1, \ldots, v_k\} \) has \( k \) vectors in \( \mathbb{R}^n \) and \( A=\langle v_1, \ldots, v_k \rangle \) Answer the following in terms of \( \text{rank}(A) \), \( k \) and \( n \).
\( S \) is independent \( \iff \)
\( S \) spans the space \( \iff \)
\( S \) is a basis \( \iff \)

Problems
4(4) For what \( a \) does the following system have
\[(a+1)x + y = 1, \quad \quad ay = a \]
(a) No solutions?
(b) Infinitely many solutions?

5(6-4) \( x = [1, 2, 3], y = [2, 3, 1], z = [3, 1, 2] \).
Write \([3, 4, 5]\) as a linear combination of \( x, y, z \).

6(4-2) What is the dimension of the space spanned by \( (1, 2, 3), (2, 3, 4) \) and \( (3, 4, 5) \)?

7(4) Give an example of a standard-form linear programming problem with two variables \( x, y \) and one inequality which has one maximum and an unbounded set of feasible solutions.

8(4) Sketch the region and find all the maxima.
\[
\begin{align*}
\text{maximize } z &= 2x + y. \\
\text{subject to } &2y - x \geq 0 \\
&y + 2x \leq 2 \\
&x, y \geq 0
\end{align*}
\]

9(6) Sketch the region and find all the minima.
\[
\begin{align*}
\text{minimize } w &= z - y - 2x \\
\text{subject to } &2x + y + z = 2 \\
&x + 2y + z = 2 \\
&x, y, z \geq 0
\end{align*}
\]
10(3) Convert to canonical form.
minimize $z = 2x + y$.
subject to $y + 2x \geq 2, \quad x \geq 0$

11(4) For the canonical problem:
Max $z = x + u + v$
subject to $x - u = 1, \quad v = 1, \quad x, u, v \geq 0$.
(a) Find a basic solution which is not feasible.
(b) Find a feasible solution which is not basic.

12(6) (Recommended problem) Translate (but don’t solve) into a general linear programming problem:
A new rose dust is being prepared by using two available products: PEST and BUG. Each kilogram of PEST contains 30 grams of carbaryl and 40 grams of Malathion, while each kilogram of BUG contains 40 grams of carbaryl and 20 grams of Malathion. The final blend must contain at least 120 grams of carbaryl and at most 80 grams of Malathion. If each kilogram of PEST costs $3.00 and each kilogram of BUG costs $2.50, how many kilograms of each pesticide should be used to minimize the cost?
State what your variables stand for. Label your constraints descriptively.

13(6) (Recommended problem) Translate (but don’t solve) into a general linear programming problem:
A coffee packer blends Brazilian coffee and Colombian coffee to prepare two products: Super and Deluxe brands. Each kilogram of Super coffee contains 0.5 kilogram of Brazilian coffee and 0.5 kilogram of Colombian coffee, while each kilogram of Deluxe coffee contains 0.25 kilogram of Brazilian coffee and 0.75 kilogram of Colombian coffee. The packer has 120 kilograms of Brazilian coffee and 160 kilograms of Colombian coffee on hand. If the profit on each kilogram of Super coffee is 20 cents and the profit on each kilogram of Deluxe coffee is 30 cents, how many kilograms of Super and how many kilograms of Deluxe coffee should be blended to maximize profit?
State what your variables stand for. Label your constraints descriptively.

14(__/4) In the tableau

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<th>x</th>
<th>y</th>
<th>z</th>
<th>u</th>
<th>v</th>
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<td>10</td>
</tr>
</tbody>
</table>

(a) What is the initial set of basic variables?

(b) The basic solution $[x, y, z, u, v, w] =$

(c) What is the entering variable?

(d) What is the departing variable?

15(__/7-8) Restate in canonical form. Solve using the Simplex Algorithm.
Write the two tableaus. Mark the entering & departing variables. List the 0-ratios. Write the final answer - either “Region empty with no max” or “Region unbounded with no max,” or “max $z = \ldots$ at $x = \ldots$.”
Don’t include slack variables in final answer.

\[
\text{min } z = -2x + 3y
\]
with
\[
2x + y \leq 6
\]
\[
x + 2y \leq 2 \quad x, y \geq 0
\]
The answer should be: max $z = 4$ at $x = 2, y = 0$. Since the answer is given, to get credit, you must show the two tableaus.