1(__/8) Solve using the two-phase method.
Omit listing θ-ratios. Arrange the slack and artifical variables so that their columns form an identity submatrix of the initial matrix.
List the 1st tableau of phase one following the initial pivoting.
List the last tableau of phase one.
List the 1st tableau of phase two following the initial pivoting.
List the last tableau of phase two.

\[
\begin{align*}
\text{min } z &= -2x - y \\
\text{with} \\
& \quad r: -2x + y \geq 1 \\
& \quad s: 2x + y \leq 5 \\
& \quad x, y \geq 0
\end{align*}
\]

Answers: 
Min \( z = \) 
Slacks: 
\( x = r = \) 
\( y = s = \) 

Dual variables and slacks 
\( r = x = \) 
\( s = y = \)
2(6) Solve without using the two-phase method.
Max \( z = 5x + 2y + u + v \)
with \( r: \ 2x + y + u + 2v \leq 6 \)
\( s: \ -3x - u \geq -15 \)
\( t: \ 5x + 4y + v \leq 24 \quad x, y, u, v \geq 0 \)

List the tableaus, all have integer values except the last tableau.
Give the optimal primal values and the dual values (remember you have to determine the sign for dual values).

3(6) For the primal problem below, state the canonical problem state the dual problem and sketch the region of feasible solutions. Calculate the slacks, the primal and the dual variables (omit the dual slacks) geometrically (draw the picture) using the Marginal Value Theorem.

Primal Problem

\[ \min z = -2x - y \]
with
\( r: -2x + y \geq 1 \)
\( s: \ 2x + y \leq 5 \)
\( x, y \geq 0 \)
4 State the dual problem.
\[
\min z = 8x + 9y \quad \text{with} \quad x \geq 0, \ y \text{ unrestricted}
\]
\[
r: 2x - 3y = 5
\]
\[
s: 4x + 8y \geq 6
\]

5 The primal objective is \( \max w = 4x + 2y + 3z \).
The dual objective is \( \min w = 12r + 10s + 10t \).
The optimal solution for the primal problem is \( x = 2, y = 0, z = 4 \) with slack \( r = 0, s = 1, t = 2 \).
Use complementary slackness and the Duality Theorem to solve the dual problem. Show your reasoning.

\[
\min w = \ldots, \text{when} \ r = \ldots, \ s = \ldots, \ t = \ldots.
\]

6 Fill in all the missing entries in the simplex tableau below. \( x, y, u \) are the primal variables; \( r, s, t \) are slacks. The first, second and third constraint constants are \( b_r = 4, b_s = 5, b_t = 6 \). The last three coefficient columns of the original tableau were an identity matrix.

<table>
<thead>
<tr>
<th></th>
<th>-1</th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>s</td>
<td>t</td>
<td>b</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>2</td>
<td>y</td>
<td>-3</td>
<td>-1</td>
<td>-2</td>
<td></td>
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<td></td>
</tr>
<tr>
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<td></td>
<td>-1</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>t</td>
<td>6</td>
<td>7</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7 Given the final tableau below. Find 4 optimal solutions.

\[
\begin{array}{cccccccc}
\text{x} & \text{y} & \text{z} & \text{r} & \text{s} & \text{t} & \text{p} \\
\hline
2 & 0 & 2 & 1 & 0 & 0 & 2 \\
1 & 1 & 0 & 0 & 1 & 0 & 1 \\
2 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 2 & 0 & 2 \\
\end{array}
\]

\[
\max w = 2 \text{ when}
\]
\[
x = \ldots, \ y = \ldots, \ z = \ldots.
\]
\[
x = \ldots, \ y = \ldots, \ z = \ldots.
\]
\[
x = \ldots, \ y = \ldots, \ z = \ldots.
\]
\[
x = \ldots, \ y = \ldots, \ z = \ldots.
\]

8 You have 100 lbs. of flour and 60 lbs. of sugar.
1 lb. of cookies uses .6 lbs. of flour, .4 lbs. of sugar.
1 lb. of candy uses 1 lb. of flour, 1 lb. of sugar.
1 lb. of crackers uses .8 lbs. of flour, .2 lbs. of sugar.
Cookies sell for $3/lb., Candy for $1/lb., Crackers for $2/lb.
Let \( k, c, r \) be the lbs of cookies, candy and crackers made. What should \( k, c, r \) be to maximize total earnings?
- State as a general linear programming problem and solve it (you may use LPSolve). Hint: \( k + c + r = 160 \).
- Someone wants to buy flour from you. How much should you charge for a lb. of flour?
- How much will your earnings increase if you get an additional 5 lbs. of sugar?
- If you can sell your crackers for $3/lb., how much will your earnings increase?
- What is the lowest price (give an integer) for candy (in dollars/lb.) which would increase your earnings?

9 Given that the objective function \( \max z = x - y \) and that the initial tableau’s constant column is \( [0; 1] \), fill in the constant columns and objective rows for (a) and (b).
Label the solution as (1) nonoptimal, (2) the unique optimal solution, (3) a nonunique optimal solution.
Warning: being able to pivot won’t necessarily guarantee a nonunique geometrically different solution.

(a)

\[
\begin{array}{cccc}
\text{x} & \text{y} & \text{r} & \text{s} \\
\hline
0 & 2 & 1 & 0 \\
1 & -1 & 0 & 4 \\
\end{array}
\]

(b)

\[
\begin{array}{cccc}
\text{x} & \text{y} & \text{r} & \text{s} \\
\hline
0 & 2 & 1 & 2 \\
1 & -1 & 0 & 4 \\
\end{array}
\]

8 Anyone wants to buy flour from you. How much should you charge for a lb. of flour?
8 How much will your earnings increase if you get an additional 5 lbs. of sugar?
8 If you can sell your crackers for $3/lb., how much will your earnings increase?
8 What is the lowest price (give an integer) for candy (in dollars/lb.) which would increase your earnings?