**Math 414  Practice Exam 5**

This practice exam contains previous problems from Practice Exam 4 for comparison purposes. One of these will be on the final.

15) There are 9 grain storage areas: a, b, c, d, e, f, g, h, i. 

a-3-c means you can directly ship one ton of grain from a to c or c to a for $30 dollars.

a-1-b, a-3-c, a-4-d, b-2-d, b-3-e, c-2-d, c-1-f, d-1-e, e-2-g, e-4-i, f-2-g, f-2-h, g-1-i, h-2-i.

Find the minimum shipping cost (in $10’s) from each node below to node a.

\[ d \rightarrow a, \quad e \rightarrow a, \quad f \rightarrow a, \quad g \rightarrow a, \quad h \rightarrow a, \quad i \rightarrow a. \]

1 point for the answer, 1 point for the graph, 3 points for correctly running the algorithm.

2(8) There are 9 grain storage areas: a, b, c, d, e, f, g, h, i.

a-3→c means you can directly ship 3 tons of grain from a to c.

a-2→b, a-6→c, a-6→d, b-4→e, c-4→d, c-4→f, d-2→b, d-6→e, e-3→g, e-4→i, f-3→g, f-3→h, g-6→i, h-2→i.

Find the maximum amount of grain which can be shipped from a to i?

1 point for the answer, 2 points for drawing the graph and listing how much should be shipped along each edge. 5 points for correctly running the algorithm.

3(7) A bowl with 20 cups of soup is passed among some of the 9 people a, b, c, d, e, f, g, i, h. It starts with “a” and stops at “i”. “a, 2→b, c, d” means “a” takes 2 cups of soup from the bowl and then passes the bowl to one of b, c, d.

\[ a_2 \rightarrow b, c, d \quad b_1 \rightarrow e \quad c_2 \rightarrow d, f \quad d_1 \rightarrow b, e \quad e_1 \rightarrow g, i \quad f_2 \rightarrow g, h \quad g_1 \rightarrow i \quad h_1 \rightarrow i \]

Find the minimum amount of soup which might be left in the bowl when it reaches i.

1 point for the answer, 4 points for modeling the problem as a transportation, assignment, max flow, shortest or longest path problem. 2 points for using the model to solve it.

4(14) Modular homes are made in plants P, Q, R, S, T and then shipped to housing sites A, B, C, D, E. Each plant makes one house and each site gets one house. The shipping costs from a plant to a site are given in the matrix. Which plants should ship to which sites in order to minimize shipping costs? Find the minimum shipping cost.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

5(16) Nails are made in plants P, Q, R, S and then shipped to housing sites A, B, C, D. The shipping costs from a plant to a site are given in the matrix. Also given are the amounts of nails (in bushels) produced by the plants and needed by the housing sites. (From Practice Exam 4).

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>A</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>C</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>D</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

What is the minimum shipping cost?

How many nails should be shipped from each plant to each housing site?

2 points for the answer. 4 points for correctly modeling the problem. 9 points for showing the steps of the appropriate algorithm.

6(6) How many steps does the following program require on a parallel computer?

\[
x := z, \\
y := z, \\
x := x+y, \\
y := x+y, \\
z := x+z, \\
x := x+z.
\]

I point for the answer. 5 points for modeling this problem as an assignment, max flow, shortest path or job scheduling problem.

7(6) Players I and II choose heads H or tails T. If both choose H, I wins $2, if both choose T, I wins $4. If they choose different sides, II wins $3.

\[
\begin{array}{c|c|c|} 
\text{I} & \text{H} & \text{T} \\
\hline 
\text{H} & p & \text{H} \\
\text{T} & 1-p & \text{T} \\
\end{array}
\]

Fill in the payoff matrix.

Find an optimal mixed strategy for player I.

What can I expect to win on average with this strategy?