Math 414  Review 1

Exam 1 next Tuesday.
Today's graded homework is placed in the bin on my office door (PSB 318) and may be picked up after 4:00 today.

Material
Lectures 1-7. No questions about SciLab or LPSolve.
The corresponding text material is on pages 1-114.
All exams are closed book: no notes, no text.
Bring a calculator, laptop and scratch paper.
Do not turn in your scratch paper; put your answers and all work you wish to show on the exam paper. Do the same with the practice exam.

Definitions
Coefficient matrix, augmented matrix, tableau and elementary row operations, tableau and reduced row echelon form, nonsingular, singular, $I$, $[A|B]$.
Basic and nonbasic variables, basic and general solutions.
Linear combination, independent, subspace spanned by..., dimension, rank($A$), convex, closed, bounded, extreme point, closed half-space, convex polyhedron, convex combination, convex hull, convex polytope, extreme point.
General linear programming problem, feasible solutions, constraints, objective function, standard form, canonical form, slack variable.
Adjacent extremes, entering and departing variables, $\theta$-ratios.

Techniques  Be able to
Use tableau and elementary row operations to transform a matrix to tableau and row reduced element forms.
Use rref to solve systems of equations and compute inverses.
Determine when a system of equations has a unique solution, infinitely many solutions or no solution.
Determine if a set of vectors is a subspace, spans a given space or is independent.
Write a vector as a linear combination of other vectors if possible.
Sketch convex polyhedra and find their extreme points.
Solve linear programming problems. This will be a major part of the test.
Convert general problems to standard and canonical form.
Translate word problems into general linear programming problems.
Solve problems using the simplex method.
You won't have to state the simplex algorithm.

Suggested Exercises
All homework exercises plus
Page Problem
9:    3bc.
20:  1, 3, 5, 7, 9, 13.
28:  5, 7, 9.
32:  3, 5.
41:  1, 3, 5, 11.
81:  1, 3, 5.
91:  1, 3, 5, 7, 12.
57:  1, 3, 5, 7, 9, 11. One or two exam problems will be a variant of one of these. Setting these up before the exam to avoid running out of time during the exam.
119: 3,5,7.

Detailed review - next page.
**Detailed review**

Be able to state the theorems below and answer questions about them. Proofs will not be required.  
*You will not be asked to prove any theorems. But you will be asked questions about the theorems.*

**Theorem.** For any linear function on a closed bounded convex set, absolute maxima and minima exist and every local maximum or minimum is also an absolute maximum or minimum.

**Theorem.** For any linear function \( f \) on a closed bounded convex set, the set of maxima is either the extreme point with the largest value among extreme points or the convex hull of all extreme points which have the largest value among extreme points. Hence finding all maxima reduces to finding the extreme points of largest value.

**Theorem.** Every general linear programming problem is equivalent to a programming problem in standard form and to one in canonical form.

**Theorem.** In the set \( S \) of feasible solutions of a canonical problem, a point is extreme iff it has more 0’s than nearby points.

**Theorem.** In a canonical problem, \( X \) is a basic solution iff \( AX = C \) and columns associated with \( X \)’s positive entries are independent.

**Lemma.** The number of basic solutions is \( \leq \binom{n}{m} \).

**Lemma.** For a canonical problem with \( n \) variables, \( m \) equations and \( k = n - m \), every basic solution has \( \leq m \) positive entries and \( \geq k \) zeros.

**Theorem.** Given a canonical problem with \( n \) variables, \( m \) independent equations, \( k = n - m \). Every extreme point is a basic solution. Hence every extreme point \( x \) has \( \geq k \) zeros.

**Fundamental Theorem.** Extreme points and feasible basic solutions are the same thing.

**Lemma.** For any set of basic variables, (a) the basic variables and the objective function can be written in terms of the parameters and a constant. (b) In the basic solution, these constants are the values of the basic variables and the objective function. (c) The basic solution is maximal iff no objective coefficient is positive. (d) If \( u \) is an entering variable which becomes a basic variable in an adjacent extreme, then \( u \) leads to an adjacent extreme with a higher \( z \) iff \( u \)’s coefficient in \( z \) is positive (thus its coefficient in the objective row is \( < 0 \)).

**Lemma.** Suppose \( p \) is an extreme and suppose \( z \) and \( p \)’s basic variables are written in terms of \( p \)’s parameters.  
(a) The departing basic variable which becomes a parameter in the adjacent extreme = the first basic variable to become 0 as entering basic variable \( u \) increases = the basic variable of \( p \) with the smallest constant/(positive coefficient) ratio.  
(b) If all constant/(positive coefficient) ratios are negative, the region is unbounded with no optimal value.