8(6/6). A cattle rancher has a single brander who takes exactly 5 seconds to brand a cow. The cows arrive at Poisson rate of 10 per minute. Find the long-term probabilities $p_0, p_1, p_2$ that there are 0, 1, or 2 cows waiting. Calculate the exact forms of $q_0, q_1$ as decimals rounded to 2 places. Warning, never use rounded decimals in calculations, save results of calculations to variables $A, B, ..$ to be used in subsequent calculation.

$$\lambda =$$  
$$\mu =$$  
$$\rho =$$  
$q_0 =$  checksum 11

$q_1 =$  checksum 22

10(3/3). Definition. For a probability distribution $p_0, p_1, p_2, \ldots$, the probability generating function is $P(z) = \sum_{n=0}^{\infty} p_n z^n = p_0 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots$

(a) Calculate $P'(0)$, i.e., the $n^{th}$ derivative at 0.

(b) Calculate $P'(1)$

(c) If $X$ is a random variable such that $P[X = i] = p_i$, write the expected value of $X$ in terms of $P(z)$. 

$$\pi_0 =$$  
$$\pi_1 =$$  
$$\pi_2 =$$
One bit of information arrives at a processor every microsecond. Note, this is the precise rate, not the expected rate. The processor requires an exponentially distributed amount of time to process the bits with rate $\ln(4)$ and hence expected service time of $1/\ln(4)$ microseconds. Find the long-term probability that 2 or fewer bits are waiting to be processed at the time of arrival of a bit.

(a) Find a simple equation for $\beta$. See last lemma. At least simplify to an equation with no summation signs. Solve it if you can. Recall that $\beta \in (0, 1)$ so $\beta = 1$ is not allowed.

(b) Suppose $\beta = 1/2$. Find the long-term probabilities that there are 0, 1, and 2 bits respectively waiting to be processed.