Mechanical systems with non-linear constraints

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Abstract

In this talk, two approaches for the description of classical mechanical systems with nonlinear constraints are analyzed: the variational and the dAlembert-Chetaev mechanics.

We start with the dAlembert-Chetaev mechanics. We prove that the dAlembert-Chetaev trajectories of a constrained mechanical system satisfy both Gauss principle of least constraint and Hölders principle. In the case of a free mechanics, they also satisfy Hertz principle of least curvature if the constraint manifold is a cone. We show that the Gibbs-Maggi-Apell (GMA) vector field (i.e. the second-order vector field which defines the dAlembert-Chetaev trajectories) conserves energy for any potential energy if, and only if, the constraint is homogeneous (i.e. if the Liouville vector field is tangent to the constraint manifold). We introduce the Jacobi-Carathodory metric tensor and prove Jacobi-Carathéodorys theorem assuming that the constraint manifold is a cone. Finally, we present a version of Liouville’s theorem on the conservation of volume for the flow of the GMA vector field.

Given a smooth Lagrangian L on the tangent bundle of the configuration space M of the constrained mechanical system, its variational trajectories are defined, through a generalization of Hamilton’s principle of stationary action, as extremals of the smooth Lagrangian functional defined on a convenient Banach manifold curves compatible with the constraint manifold. In the particular case of a Lagrangian given by the positive definite quadratic form induced by a metric tensor on M, this amounts to a generalization of sub-Riemannian geometry. Among the main results, it is proven that, under a regularity condition on the Lagrangian L, the normal extremals of the Lagrangian functional are given by the projections on M of a Hamiltonian vector field defined on the generalized mixed bundle W.