(15) **1. a)** State Wilson’s Theorem.
   **b)** Illustrate the idea of the proof of Wilson’s Theorem by using the example $p = 13$.

(20) **2. a)** Find the smallest integer $n$ such that $n \geq 8$ and $n \equiv 7 \pmod{4}$,
   $n \equiv 7 \pmod{5}$, and $n \equiv 7 \pmod{13}$.
   **b)** Given that $12x \equiv 1 \pmod{59}$ has $x = 5$ as a solution, solve each of the following:
   i) $12x \equiv 8 \pmod{59}$
   ii) $12x \equiv 13 \pmod{59}$
   iii) $12x \equiv 35 \pmod{59}$.

(15) **3.** Let $a$ and $b$ be positive integers. Prove that there exist $x$ and $y$ with $a = (x, y)$,
   $b = [x, y]$ if and only if $a|b$.

(25) **4. a)** Prove that the Diophantine equation $ax + by = c$ has a solution if and only if
   the congruence $ax \equiv c \pmod{b}$ has a solution.
   **b)** Let $g = (a, b)$. By Theorem 1.4, $ax + by = g$ has a solution.
   Use this to prove that $ax + by = c$ has a solution if and only if $g|c$.
   (DO NOT use theorems about congruences.)

(25) **5. a)** Prove that for any $n$, $n^9 - n$ is divisible by 10.
   **b)** Prove that $n^9 - n$ is divisible by 20 if and only if $n$ does not have the form $4k + 2$. 