Start with a curve given parametrically in the \(xz\)-plane, viz.\[
\beta(v) = (x(v), 0, z(v)) = (\varphi(v), 0, \psi(v)).
\]
Revolving this curve around the \(z\)-axis yields a surface which we can describe in cylindrical coordinates as \(r = \varphi(v),\ z = \psi(v)\). To make things cleaner, we can let \(R(u)\) denote a unit vector with angle \(u\) in the \(xy\)-plane and let \(k\) be the usual unit vertical vector. Thus
\[
R(u) = (\cos u, \sin u, 0), \quad k = (0, 0, 1).
\]
Note that \(R_u(u) = (-\sin u, \cos u, 0)\) and \(R_u \perp R\) and \(R_u \perp k\). (\(R_u\) is simply the tangent vector to the unit circle in the \(xy\)-plane.) Then the surface \(S\) is parametrized by
\[
\mathbf{X}(u, v) = \varphi(v)R(u) + \psi(v)k
\]
\[
= (\varphi(v) \cos u, \varphi(v) \sin(u), \psi(v)).
\]
Now \(\mathbf{X}_u = \varphi'(v)R(u) + \psi'(v)k\), and so the vector \(\psi'(v)R(u) - \varphi'(v)k\) is orthogonal to \(\mathbf{X}_v\). But we also note that it is orthogonal to \(\mathbf{X}_u = \varphi(v)R_u\), because \(R_u \perp R\) and \(R_u \perp k\). Thus we get a unit normal to \(S\) by taking
\[
\mathbf{N} = \frac{\psi'(v)R(u) - \varphi'(v)k}{\sqrt{\varphi'(v)^2 + \psi'(v)^2}}.
\]
Now notice that \(\mathbf{N}\) is perpendicular to \(R_u\), and \(R_u\) is independent of \(v\). Thus \(\mathbf{N}_v\) is also perpendicular to \(R_u\), and so of course also \(\mathbf{N}_v \perp \varphi(v)R_u = \mathbf{X}_u\). Of course we also know that \(\mathbf{N}_v \perp \mathbf{N}\). It follows (since we know that \(\mathbf{X}_u, \mathbf{X}_v,\) and \(\mathbf{N}\) are at right angles to each other) that \(\mathbf{N}_v = d\mathbf{N}_p(\mathbf{X}_v)\) is in the direction of \(\mathbf{X}_v\). I.e. \(\mathbf{X}_v\) is an eigenvector for \(d\mathbf{N}_p\). So the meridians of the surface are lines of curvature. Since \(\mathbf{X}_u\) is orthogonal to \(\mathbf{X}_v\), we also see that \(\mathbf{X}_u\) is in the other principal direction, so the parallels are also lines of curvature. (In fact, \(\mathbf{N}_u = \psi' \mathbf{X}_u / \varphi \sqrt{\varphi'(v)^2 + \psi'(v)^2}\).)
Now a parallel is a horizontal circle with radius \( \varphi(v) \), and so its curvature is \( 1/\varphi(v) \) and its normal vector is the horizontal vector \( \mathbf{n} = -\mathbf{R}(u) \). To obtain the normal curvature of \( S \) in the direction \( \mathbf{X}_u \), we need to multiply \( 1/\varphi(v) \) by \( \langle \mathbf{N}, \mathbf{n} \rangle \). This yields

\[
k(\mathbf{X}_u) = \frac{-\psi'(v)}{\varphi(v) \sqrt{\varphi'(v)^2 + \psi'(v)^2}}.
\]

Since \( |\mathbf{X}_u|^2 = \varphi(v)^2 \), this yields

\[
e = \frac{-\psi'(v)\varphi(v)}{\sqrt{\varphi'(v)^2 + \psi'(v)^2}}.
\]

On the other hand, we have seen that the normal vector for the meridians is the same as the normal vector \( \mathbf{N} \) for the surface. And the curvature of the meridians \( k(\mathbf{X}_v) \) is the same as the curvature of the original curve \( \beta \), and from basic calculus this is

\[
\frac{\varphi'\psi'' - \varphi''\psi'}{(\varphi'(v)^2 + \psi'(v)^2)^{3/2}}.
\]

Since \( |\mathbf{X}_v|^2 = \varphi'(v)^2 + \psi'(v)^2 \), this gives

\[
g = \frac{-\varphi'\psi'' + \varphi''\psi'}{\sqrt{\varphi'(v)^2 + \psi'(v)^2}}.
\]

And finally, \( f = -\langle \mathbf{N}_u, \mathbf{X}_v \rangle = 0 \).