Math 471 Supplemental Homework 1 – Fall 2017

Due: September 14

1 Basic definitions

1. A point is selected at random from unit disk \( \{(x, y) : x^2 + y^2 \leq 1\} \). What is the probability that the point is at least 1/2 unit from the origin?

2. TURN THIS ONE IN. A box has 10 balls numbered 1 through 10. A ball is picked at random from this box, then another is selected at random from the remaining 9 balls. Find the probability that the numbers on the two balls differ by at least 2.

3. Two standard 6-sided dice are rolled; what is the probability that the resulting throw (i.e., the sum of the spots on the uppermost sides) is an even number?

4. TURN THIS ONE IN. Find \( \alpha \) so that \( \mathbb{P} \) is a measure, where:

\[ \Omega = \{1, 2, 3\}; \mathcal{F} = \mathcal{P}(\Omega); \mathbb{P}(\{1\}) = 1/3, \mathbb{P}(\{2\}) = 2/6, \mathbb{P}(\{3\}) = \alpha \]

What is \( \mathbb{P}(\{2, 3\}) \)?

5. Let \( \Omega = \mathbb{R}^2 \), and \( \mathcal{B} \) be the smallest \( \sigma \)-algebra containing all products of the form \( I \times J \), where \( I \) and \( J \) are intervals (open or closed or half-open) in the real line. (a) Show that any horizontal or vertical line in \( \Omega \) is in \( \mathcal{B} \). (b) Is the line \( \{(x, y) : y = x\} \) in \( \mathcal{B} \)? Why or why not?

6. TURN THIS ONE IN. Let \( (\Omega, \mathcal{F}, \mathbb{P}) \) be a probability space, and \( A_0, A_1, A_2, \ldots, A_n, \ldots \) \( (n \in \mathbb{N}) \) be events in \( \mathcal{F} \) (not necessarily disjoint!). Show that

\[ \mathbb{P}(\bigcup_n A_n) \leq \sum_n \mathbb{P}(A_n) \]

2 Bayes and conditional probability

7. If \( B \subseteq A \) and \( P(B) \neq 0 \) then \( P(A|B) = 1 \)
8. **TURN THIS ONE IN.** You have two coins, a fair one ($P(\text{heads}) = .5$) and an unfair one ($P(\text{heads}) = 1/3$); the coins are otherwise identical. You take one of these coins from your pocket and toss it; it falls heads up. How likely is it you chose the fair coin?