p. 812 # 25. Find the curvature of the path given parametrically for $\mathbf{r}(t) = t\,\mathbf{i} + \frac{1}{2}t^2\,\mathbf{j} + \frac{1}{3}t^3\,\mathbf{k}$ and find the tangential and normal components of acceleration.

$$\begin{aligned} \mathbf{r}(t) &= t\,\mathbf{i} + \tfrac{1}{2}t^2\,\mathbf{j} + \tfrac{1}{3}t^3\,\mathbf{k} \\ \mathbf{v} = \mathbf{r}'(t) &= \mathbf{i} \ + t\,\mathbf{j} \quad + t^2\,\mathbf{k} \\ \mathbf{a} = \mathbf{r}''(t) &= \qquad\quad \mathbf{j} \quad + 2t\,\mathbf{k} \end{aligned}$$

$$\text{speed} = \frac{ds}{dt} = ||\mathbf{v}|| = \sqrt{t^4 + t^2 + 1}$$

$$a_{\mathbf{T}} = \frac{d^2s}{dt^2} = ||\mathbf{v}||' = \frac{\frac{1}{2}(4t^3 + 2t)}{\sqrt{t^4 + t^2 + 1}} = \frac{2t^3 + t}{\sqrt{t^4 + t^2 + 1}}$$

$$\mathbf{T} = (\,\mathbf{i} + t\,\mathbf{j} + t^2\,\mathbf{k})/\sqrt{t^4 + t^2 + 1}$$

$$\frac{d^2s}{dt^2}\mathbf{T} = \frac{2t^3 + t}{t^4 + t^2 + 1}\,(\,\mathbf{i} + t\,\mathbf{j} + t^2\,\mathbf{k})$$

$$\begin{aligned} k\left(\frac{ds}{dt}\right)^2\mathbf{N} &= \mathbf{a} - \frac{d^2s}{dt^2}\mathbf{T} \\ &= \frac{(-2t^3 - t)\,\mathbf{i} + (-t^4 + 1)\,\mathbf{j} + (t^3 + 2t)\,\mathbf{k}}{t^4 + 2t + 1} \end{aligned}$$

$$\begin{aligned} a_{\mathbf{N}} = k\left(\frac{ds}{dt}\right)^2 &= ||k\left(\frac{ds}{dt}\right)^2\mathbf{N}|| \\ &= \frac{\sqrt{(-2t^3 - t)^2 + (-t^4 + 1)^2 + (t^3 + 2t)^2}}{t^4 + t^2 + 1} \\ &= \frac{\sqrt{t^8 + 5t^6 + 6t^4 + 5t^2 + 1}}{t^4 + t^2 + 1} \\ &= \frac{\sqrt{(t^4 + t^2 + 1)(t^4 + 4t^2 + 1)}}{t^4 + 4t^2 + 1} \\ &= \frac{\sqrt{t^4 + 4t^2 + 1}}{\sqrt{t^4 + t^2 + 1}} \end{aligned}$$

$$k = \frac{a_{\mathbf{N}}}{(ds/dt)^2} = \frac{(t^4 + 4t^2 + 1)^{\frac{1}{2}}}{(t^4 + t^2 + 1)^{\frac{3}{2}}}$$